Notes on Induction, Probability and Confirmation

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The Problem of Induction

The problem of induction has been taken to be the problem of justifying the inference from the observed to the unobserved; or from particular instances to generalisations; or from the past to the future. The problem of the *rational grounds for induction* came in sharp focus in Hume's work. What is known as his scepticism about induction is the claim that any attempt to infer, based on experience, that a regularity that has held in the past *will* or *must* continue to hold in the future will be circular and question-begging.

John Venn

Venn claimed that 'it is regular sequence of some kind or other which constitutes the whole logical significance of causation', and that it was Hume who shifted the signification of cause and effect to the new track of regularity. For instance, Venn argues, Locke took this regularity for granted without asserting it explicitly, whereas Hume did exactly the latter.

Venn distinguishes between Hume's assertion of the regularity and his question concerning the foundation of our belief in such a regularity. This distinction enables Venn to associate the problem of causation with the problem of induction. For, as he says, to understand the causal relation in a 'wide sense', 'seems nearly equivalent to assigning as the ultimate ground for an induction the observed Uniformity of Nature' (1889, 128). What he seems to mean by this 'wide sense' of causation is the regularity of the constant conjunction between two events. Hence, when Hume speaks of the observation of a constant conjunction, he refers to the uniformity of nature.

Venn takes Hume's answer to the question of the foundation of our belief in regularity to imply that by no process of reasoning is one is entitled to draw a conclusion concerning an the occurrence of a past association in the future (op.cit., 131). For Venn there is no logical solution to the problem of uniformity: 'It must be assumed as a postulate, so far as logic is concerned, that the belief in the Uniformity of Nature exists, and the problem of accounted for it must be relegated to psychology' (op.cit., 132).

Bertrand Russell

Russell's 'principle of induction': 'The principle of induction as applied to causation, says that, if A has been found very often accompanied or followed by B, and no instance is known of A not being accompanied or followed by B, then it is probable that on the next occasion on which A is observed it will be accompanied or followed by B'.

So the Principle asserts the following: (1) the greater the number of cases in which A has been found associated with B, the more *probable* it is that A is always associated with B (if no instance is known of A not associated with B); (2) a sufficient number of cases of association between A and B will make it nearly certain that A is always associated with B.

Thus stated, the Principle of Induction *cannot* be refuted by experience, even if an A is actually found *not* to be followed by B. But neither can it be proved on the basis of experience. Russell's claim was that without a principle like this, science is impossible and that this principle should be accepted on the ground of its intrinsic evidence.

John Maynard Keynes

According to Keynes, though Hume's sceptical claims are usually associated with causation, the real object of his attack is *induction* i.e., the inference from past particulars to future generalizations. (1921, 302).

Keynes's argument is the following:

(1) A constant conjunction between two events has been observed in the past. This is a fact. Hume does not challenge this at all.
(2) What Hume challenges is whether we are justified to infer from a past constant conjunction between two events that it will also hold in the future.
(3) This kind of inference is called *inductive*.
(4) So, Hume is concerned with the problem of induction.

As he says: 'Hume showed, not that inductive methods were false, but that their validity had never been established and that all possible lines of proof seemed equally unpromising' (ibid.).

Keynes's answer to the problem of induction

(1) The principle of the limited variety of a finite system.
Suppose that although *C* has been invariably associated with *E* in the past, there is an unlimited variety of properties $E_1, ..., E_n$ such that it is logically possible that future occurrences of *C* will be accompanied by any of the E_i's, instead of *E*. Then, and if we let *n* (the variety index) tend to infinity, we cannot even start to say how likely it is that *E* will occur given *C*, and the past association of *C*s with *E*s. The Principle of Limited Variety excludes the possibility just envisaged. According to this Principle, there are no infinitely complex objects; alternatively, the qualities of an object cannot fall into an infinite number of independent groups. For Keynes, the qualities of an object are determined by a finite number of primitive qualities; the latter (and their possible combinations) can generate all qualities of an object. Given this, any newly observed quality does not alter the probability of a certain generalization since it is reduced to the already observed.

(2) A generalization of the form 'All Cs are Es' should be read thus 'It is probable that any given C is E' rather than thus 'It is probable that all Cs are Es. So, the issue is the next instance of the observed regularity and not whether it holds generally (op.cit., 287-288).

(3) To avoid the charge of circularity, Keynes distinguishes between the inductive hypothesis and the inductive method.
(a) Inductive hypothesis: it is the absolute assertion of the finiteness of the system under consideration. Obviously, in such a system the principle of limited variety holds (op.cit. 282&285).

(b) Inductive method: p(c/h&e)> p(c/h), i.e., the process of strengthening a conclusion c grounded on a hypothesis h, by taking into account the evidence e.

Keynes claims that it is not circular to use the inductive method to strengthen the inductive hypothesis itself, relative to some more primitive and less far-reaching assumption. That is to say, we have a reason to assign an a priori probability to the inductive hypothesis H. Then by appealing to experience (and its actual a posteriori conformity with what is asserted by H), the probability of H is raised by using the inductive method. This inductive hypothesis is neither a self-evident logical axiom (op.cit., 291) nor an object of direct acquaintance (op.cit., 294). But nevertheless, he insists that it is true of some factual systems (op.cit., 291).

Hans Reichenbach

According to Reichenbach, we can dispense with the principle of the uniformity of nature if we accept an assumption which states that the limit of a relative frequency exists. He says: 'If the inductive conclusion is recognized as being asserted, not as a statement maintained true or probable, but as an anticipative posit, it can be shown that a uniformity postulate is not necessary for the derivation of the inductive conclusion' (1949, 473). Then he postulates the 'inductive rule', being a rule for the derivation of statements about the future from statements for the past. That rule is a necessary and sufficient condition for finding the limit of a relative frequency, if there is indeed such a limit:

If there is a limit of the relative frequency, then to use the inductive rule will be a sufficient condition for finding it in a desired degree of approximation.

If we do not know whether there is such a limit, then, if such a limit exists, the rule of induction is a necessary condition for finding it.

So, Reichenbach takes it that he found a solution to Hume's problem:
(1) Hume is right in asserting that the conclusion of the inductive inference cannot be proved to be true or even probable.
(2) But Hume is wrong in stating that inductive inferences are unjustifiable. They can be justified as an instrument that realises the necessary conditions of prediction, to which we resort because sufficient predictions are beyond our reach. (op.cit. 475).

As he puts it: 'The rule of induction is justified as an instrument of positing because it is a method of which we know that if it is possible to make statements about the future we shall find them by means of this method' (ibid.).

The Mathematical Theory of Probability

The theory of Probability is a mathematical theory first introduced in the seventeenth century in connection with games of chance and fully axiomatised by the Russian mathematician Andrei Nikolaevich Kolmogorov (1903-1987) in his *Foundations of the Theory of Probability* (1933). Apart from its use in the sciences, probability theory has become very important to the philosophy of science especially in relation to the theory of confirmation and the problem of induction.

Suppose there is an experimental model M, and a sample of space S of possible outcomes (events): $\{A_1,\dots, A_n\} \in S$

$P(A_i)$ is the probability of an event A_i in S

A probability function on S (P: S $\rightarrow$ [0,1]) is an assignment of real numbers $P(A_i)$ which satisfy the following axioms

1. For any outcome A_i, $P(A_i) \geq 0$
2. $P(S)=1$
3. For any infinite sequence of disjoint events $A_1, \ldots, A_n$
4. $P(\cup_{i=1 \text{ to } \infty} A_i) = \Sigma_{i=1 \text{ to } \infty} P(A_i)$

Theorems

$P(\varnothing)=0$

For any finite sequence of disjoint events $A_1, \ldots, A_n$

$P(\cup_{i=1 \text{ to } n} A_i) = \Sigma_{i=1 \text{ to } n} P(A_i)$ For any event A, $P(A^C) = 1 - P(A)$

For any event A, $0 \leq P(A) \leq 1$

For any two events A and B, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Independent events: Two events A and B are independent if the occurrence or non-occurrence of one of them has no influence on the occurrence or non-occurrence of the other.

Two events A and B are independent if $P(A \cap B) = P(A)P(B)$.

n events $A_1, \ldots, A_n$ are independent if

$P(A_1 \cap A_2 \cap \ldots \cap A_n) = P(A_1)P(A_2)\ldots P(A_n)$

Conditional probabilities tell us how the probability of an event A is affected by the occurrence of another event B.

The occurrence of B reshapes the sample space B

If A and B are two events such that $P(B) > 0$, then: $P(A/B)=P(A \cap B)/P(B)$

Example: What is the probability that a card drawn from a deck is an Ace given that it is black?

P(Ace/Black)=P(Ace & Black)/P(Black)= (2/52)/(1/2)=2/13

Theorems for conditional probabilities

For any events A and B, if P(A/B) is defined, then $0 \leq P(A/B) \leq 1$

If A is a subset of B, then P(A/B)=1

If A and B are two mutually exclusive events, then: $P(A \cup B/C)=P(A/B)+P(B/C)$

(Multiplication rule): $P(A \cap B/C)=P(A/C)P(B/C \cap A)$

If And B are independent events given C, then $P(A \cap B/C)=P(A/C) + P(B/C)$

P(-B/A)=1-P(B/A)

Since we shall be dealing with **propositions**, it is convenient to reformulate the axioms so that they hold for propositions.

An algebra is a collection of propositions such that for any two propositions p, q in the collection

p & q

$p \vee q$

-p

-q

are in the collection.

T is a tautology

A probability function on an algebra is an assignment of real numbers P(h) which satisfy the following axioms

5. For any proposition h, $P(h) \geq 0$
6. $P(T)=1$
7. If h_1 and h_2 are inconsistent, then $P(h_1 \vee h_2) = p(h_1) + p(h_2)$

Theorems

$P(\perp)=0$

For any finite sequence of mutually inconsistent propositions $h_1, \ldots, h_n$

$P(\vee_{i=1 \text{ to } n} h_i) = \Sigma_{i=1 \text{ to } n} P(h_i)$ For any proposition h_i, $P(-h_i) = 1 - P(h_i)$

For any proposition h_i, $0 \leq P(h_i) \leq 1$

For any two propositions h_1 and h_2, $P(h_1 \vee h_2) = P(h_1) + P(h_2) - P(h_1 \& h_2)$

Probabilistic Independence:

E is independent of H iff $P(h\&e) = P(h) P(e)$.

Equivalently: $P(h/e)=P(h)$ (Informational irrelevance)

Proof: Suppose $P(h/e)=P(h)$

$P(h\&e) = P(h/e) P(e)=P(h) P(e)$

Suppose $P(h\&e) = P(h) P(e)$

$P(h/e) P(e)=P(h) P(e)$

$P(h/e)=P(h)$

Definition of conditional probability

$P(h/e)=P(h\&e)/P(e)$, if $P(e) > 0$

Theorems for conditional probabilities

For any propositions A and B, if P(A/B) is defined, then $0 \leq P(A/B) \leq 1$

If A logically entails B, then $P(A/B)=1$

If A and B are two mutually exclusive, then: $P(A \vee B/C)=P(A/B)+P(B/C)$

(Multiplication rule): $P(A\&B/C)=P(A/C)P(B/C\&A)$

If A and B are independent given C, then $P(A\&B/C)=P(A/C) + P(B/C)$

$P(-B/A)=1-P(B/A)$

Bayes's theorem

$$P(h/e) = \frac{P(e/h) \cdot P(h)}{P(e)}$$

Proof:

$P(e/h)=P(e\&h)/P(h)$

$P(e/h)P(h)=P(e\&h)$

$P(h/e)=P(h\&e)P(e)$

$P(h/e)=P(e/h)P(h)/P(e)$

Theorem of total probability

$P(e)= \Sigma_i P(e/h_i)P(h_i)$, where h_i's are mutually exclusive and exhaustive.

Special case:

$P(e)=P(e/h)P(h)+P(e/-h)P(-h)$

Alternative formulation of Bayes's Theorem

$$P(h/e)=\frac{P(e/h)\cdot P(h)}{P(e/h)\cdot P(h)+P(e/-h)\cdot P(-h)}$$

Issues in the Interpretation of Probability

Though there has been little disagreement in relation to the mathematical formalism, there has been considerable controversy regarding the interpretation of the formalism—and in particular the *meaning* of the concept of probability.

There have been two broad strands in understanding probability: an epistemic and a physical. According to the first, probability is intimately connected with knowledge or belief in that it expresses degrees of knowledge, or degrees of belief, or degrees of rational belief. According to the second strand, probabilities, like masses and charges, are objective features of the world.

The epistemic strand is divided into two camps according to whether priori probabilities express rational (objective) or merely subjective degrees of belief. Both camps agree that the probability calculus is a kind of an extension of ordinary deductive logic, but the subjectivists deny that there are logical or quasi-logical principles (like the principle of indifference) which ought to govern the rational distribution of prior probabilities.

The physical strand is divided into two camps according to whether there can be irreducible single-case probabilities (or chances). The advocates of the view that probabilities are relative frequencies take the concept of probability to be meaningful only if it is applied to a collective of events, while the advocates of chances take it to be meaningful that probabilities can be attributed to single unrepeated events. Historically, the epistemic conception of probability came first, as exemplified in the classical interpretation, and the conception of physical probability was developed as a reaction to the continental rationalism of Laplace and his followers. Richard von Mises (1883-1953) who was one of the founders of the view that probabilities are limiting relative frequencies, argued that probability theory was an empirical science (like mechanics and geometry) which deals with mass phenomena (e.g., the behaviour of the molecules of a gas) or repetitive events (e.g., coin tosses). He then tried to develop the theory of probability on the basis of empirical laws, viz., the law of stability of relative frequencies and the law of randomness. As we shall see later on, Carnap aimed to bring together the epistemic and the physical strands under his two-concept view of probability.

Epistemic Accounts

The Classical Conception

The classical conception of probability, advocated by most of the founders of the probability calculus and most notably by Bernoulli and Laplace, defines probability as the ratio of favourable to equally possible instances. For instance, the probability that a fair coin will land tails in a toss is the ratio of the number favourable instances (tails) over the number of all equally possible instances (heads and tails); that is, it is one half. The classical interpretation takes it that the probabilities are measures of *ignorance*, since equal possibility of occurrence is taken to imply that there is no reason to favour one possible outcome over the others. If, for instance, we know that exactly one of three mutually exclusive and exhaustive events A, B and C will (or must) occur, but we have no idea which one (or no reason to expect one more than the others), then the probability of each of them is one third. The principle that operates behind the classical interpretation is the Principle of Indifference (see below). Given this, the classical interpretation can be seen as a species of the logical interpretation, since it takes it that probabilities express degrees of credibility or degrees of rational certainty.

The Logical Conception

The logical interpretation of probability conceives of the probability calculus as a branch of logic, and of probability itself as a logical relation that holds between propositions. This logical relation is taken to be a relation of partial entailment—for instance, it is said that though the proposition *p* does not (deductively) entail the conjunction (*p* and *q*), it entails it partially, since it entails one of its conjuncts (namely *p*). The probability calculus is then used to calculate the probability of a proposition (say a hypothesis) in relation to another proposition (say, a proposition expressing the evidence) that partially entails it. On this approach, the degree of partial entailment is the degree of rational belief. That is, it is the degree of belief a rational agent ought to have in the truth of the hypothesis in light of the evidence that confirms it. This interpretation was defended by Keynes and Carnap. Keynes claimed that rational agents possess a kind of logical intuition by means of which they *see* the logical relation between the evidence and the hypothesis. But Frank Ramsey famously objected to Keynes' claim that he could not see these logical relations and that he expected to be persuaded by argument that they exist. As we shall see in detail below, Carnap developed the logical interpretation into a quantitative system of inductive logic, which relied on the principle of indifference for the assignment of prior probabilities, that is of prior rational degrees of beliefs. (Because of its current popularity, we shall deal with the subjective conception of probability in detail below.)

Physical Accounts

The Relative Frequency Conception

According to this frequency interpretation of probability, probability is about the relative frequency of an attribute in a collective. As von Mises (one of the founders of this interpretation) put it, first the collective, then the probability. A collective is a large group of repeated events, e.g., a sequence of coin-tosses. Suppose that in such a collective of *n* tosses, we have *m* heads. Then, the *relative frequency* of heads is m/n. Probability is then defined as limiting relative frequency, that is the limit of the relative frequency m/n, as *n* tends to infinity.

$$P(E)=\lim_{n\to\infty} m/n$$

A consequence of this account is that either probabilities cannot be meaningfully applied to single events or to attribute some probability to a single event is to transfer a probability associated with an infinite sequence to a member of this sequence.

However, being properties of sequences of events, probabilities become fully objective. There is no guarantee, of course, that the limit of the relative frequency exists. That relative frequencies converge in the limit is a postulate of the frequency interpretation. It is noteworthy that even if the limit of the relative frequency does exist, any finite relative frequency might well be arbitrary far from the limiting relative frequency. Hence, making short-term predictions on the basis of actual relative frequencies is very hazardous. What is guaranteed is that *if* the limit of the relative frequency does exist, there will be convergence of the actual relative frequencies to it in the long run. This fact has been used by Reichenbach (another leading advocate of the frequency interpretation) in his attempt to offer a pragmatic vindication of induction—and in particular of the straight rule of induction. It turns out, however, that this property of convergence is possessed by any of a class of asymptotic rules, which nonetheless yield very different predictions in the short run.

The Propensity Conception

Based on the problem of single-case probabilities, Popper developed the propensity interpretation of probability, according to which probabilities are objective properties of single (and unrepeated) events. In the version defended by Popper, propensities are properties of experimental conditions (chance set-ups). Hence, a fair coin does *not* have an inherent propensity one half to land heads. If the tossing takes place in a set-up in which there are slots on the floor, the propensity of landing heads is one third, since there is the (third) possibility that the coin sticks in the slot. This account of probability is supposed to avoid a number of problems faced by the frequency interpretation. In particular, it avoids the problem of inferring probabilities in the limit. But, especially in Popper's version, it faces the problem of specifying the conditions on the basis of which propensities are calculated.

Given that an event can be part of widely different conditions, its propensity will vary according to the conditions. Does it then make sense to talk about *the* true objective singular probability of an event? Even if this problem is not taken seriously (after all, the advocate of propensities may well argue that propensities are the sort of thing that varies with the conditions), it has been argued that probabilities cannot be identified with propensities. The problem is this. There are the so-called inverse probabilities, but it does not make sense to talk about inverse propensities. Suppose, for instance, that a factory produces red socks and blue socks and uses two machines (Red and Blue) one for each colour. Suppose also that some socks are faulty and that each machine has a definite probability to produce a faulty sock, say one out of ten socks produced by the Red machine are faulty. We can meaningfully say that the Red machine has an one tenth propensity to produce faulty socks. But we can also ask the question: given an arbitrary faulty sock, what is the probability that it has been produced by the Red machine? This question is meaningful and has a definite answer. But we cannot make sense of this answer under the propensity interpretation. We cannot meaningfully ask: what is the propensity of an arbitrary faulty sock to have been produced by the Red machine?

Carnap's Inductive Logic: A Presentation

Explication of concepts as a philosophical task.

1. Explication of the pre-scientific concept of probability.

Carnap distinguished between two explicata for the *explicandum* 'probability'. The first, what he calls 'probability(1)', is logical probability which interprets the probability calculus as stating (rational) degrees of beliefs in the certainty of propositions. The second, what he calls 'probability(2)', is the so-called objective probability, which identified probability with the relative frequency of an event in a certain sample.

The logical probability is meant to capture the *degree of confirmation* of a hypothesis in the light of some evidence.

2. Carnap makes ‘degree of confirmation’ the basic concept in his programme. There are actually three (semantic) concepts:
classificatory: ‘e confirms or supports h’;
comparative: ‘e confirms h_1 more strongly than e’ confirms h_2’; and,
quantitative: the degree of confirmation of h by e is r; or to put it symbolically, $c(h/e) = r$, where ‘r’ stands for a real number.

While Keynes was in favour of a comparative view, Carnap concentrates his attention on establishing a quantitative concept and on defining an adequate measure-function.

3. The **degree of confirmation** must satisfy the following desiderata:
it must be a *computable measure function* which expresses the degree of confirmation of a hypothesis in light of certain evidence;
it must be justified a priori;
and it must be applicable to the language of science;

4. Carnap begins with the construction of the **language** in which inductive logic is to be applied. His aim is that inductive logic be adequate for application to the comprehensive language of sciences.

Language systems: L_∞, involving one infinite system and L(N), involving finite systems and forming an infinite sequence of systems with N running through all positive integers: L_1,L_2,L_3, etc. The system L_∞ contains an infinite sequence of individual constants, which refer to all the individuals in its domain of discourse. It contains a finite number of primitive predicates of any degree. Those of degree one, e.g. ‘P_1’,’P_2’, etc. designate properties of individuals and those with degree two, e.g. ‘R_1’, ‘R_2’, etc. designate dyadic relations between individuals. Every system, either finite or infinite, contains an infinite number of individual variables e.g., ‘x_1’, ‘x_2’, ‘x_3’ etc. The systems contain universal quantifiers with individual variables, e.g., ‘(x)(Px)’, the sign ‘=’ as the customary sign of identity, the sign ‘t’ as a tautological sentence. Of the connectives, they contain the signs of negation (‘-‘), of disjunction (‘$\vee$’) and of conjunction (‘&’)

5. Important definitions:
Molecular Q-predicates = Conjunctions in which every primitive predicate or its negation occurs. For instance, if there are only three primitive predicates P_1, P_2, P_3 then there are 2^3 (=8) Q-predicates.

Example
Let us envisage a simple weather-world which has three basic traits, i.e., hot (H), rainy (R) and windy (W). Then, there are the following 8 molecular predicates:

	Hot (H)	Rainy (R)	Windy (W)
Q1	H	R	W
Q2	H	R	-W
Q3	H	-R	W

Q_4	H	-R	-W
Q_5	-H	R	W
Q_6	-H	R	-W
Q_7	-H	-R	W
Q_8	-H	-R	-W

Each predicate expression of the language can be transformed into an equivalent expression in terms of the molecular predicates Q_is. For example,

H &R → $Q1 \vee Q2$

H & -R → $Q3 \vee Q4$

-H & -W → $Q6 \vee Q8$

R $\vee$W → $Q1 \vee Q2 \vee Q3 \vee Q5 \vee Q6 \vee Q7$

State Description = a class of sentences which describe *completely* a possible state of the domain of individuals of L (i.e., a possible complete state of affairs) with respect to all the attributes (i.e., properties and relations). It consists of the conjunction of either i or -i but not both of them, of *each atomic sentence* i in the system L.

Example

Imagine a language with three individual constants and one predicate F. The 8 state descriptions are:

D1. Fa_1 & Fa_2 & Fa_3

D2. Fa_1 & Fa_2 & $-Fa_3$

D3. Fa_1 & $-Fa_2$ & Fa_3

D4. Fa_1 & $-Fa_2$ & $-Fa_3$

D5. $-Fa_1$ & Fa_2 & Fa_3

D6. $-Fa_1$ & Fa_2 & $-Fa_3$

D7. $-Fa_1$ & $-Fa_2$ & Fa_3

D8. $-F\ a_1$ & $-Fa_2$ & $-Fa_3$

Range of a sentence = the class of those state descriptions in which the sentence holds true.

Example: the range of Fa_1 is the set {D1, D2, D3, D4}

The range of $Fa_1 \vee Fa_2$ is the set {D1, D2, D3, D4, D5, D6}

The range of Fa_1& Fa_3 is {D1, D2}

L-concepts = logical concepts; these concepts (i.e., L-true, L-false, L-implication etc.) are defined on the basis of the foregoing definitions. For instance, L-true is a concept with a universal range, i.e., if it holds in every state description; L-implication is logical entailment.

The requirement of completeness of language: the set of the primitive predicates of the language must be sufficient for expressing *every qualitative attribute* of the individuals in the universe of discourse of L.[1] Later on Carnap, abandoned this

[1] This requirement seemed absolutely necessary for the Carnapian system, since the language systems affect the c-values in the theory of inductive logic. For the time being, all we need to stress is that this requirement implies that a language L *mirrors* its universe of discourse. Whatever there is within it can be exhaustively expressed within L. Here is Carnap's example. Take a language L with only two predicates, P_1 and P_2 interpreted as Bright and Hot. Then every individual in the universe of discourse

requirement, and he replaced it with the following: The value of the confirmation function c(h/e) remains unchanged if further families of predicates are added to the language (RAE, 975). According to this requirement, the value of c(h/e) depends *only* on the predicates occurring in *h* and *e*. Hence, the addition of new predicates to the language does not affect the value of c(h/e). This new idea amounts to what Lakatos called the *minimal language requirement* (Lakatos 1968, 325), according to which the degree of confirmation of a proposition depends only on the minimal language in which the proposition can be expressed.

The requirement of total evidence: in the application of inductive logic to a given situation, the total evidence available must be taken as the basis for determining the degree of confirmation of a hypothesis.

Isomorphic expressions = one is constructed from the other by replacing individual constants with others. In other words, they are conjunctions which differ from each other in the order of the conjuncts.

Structure Description corresponding to a class of state descriptions = the disjunction of all the isomorphic state descriptions. Two isomorphic state descriptions have the same *structure*. Simply put, a structure description is the disjunction of all state descriptions *with the same number of unnegated predicates*. If we imagine a language L with one primitive predicate A, there are two molecular predicates Q: Q_1=F and Q_2= -F. The class of state descriptions of the n+1 individuals in that language is: $\pm F(a_1)\&\pm F(a_2)\&...\&\pm F(a_{n+1})$ and consists of 2^{n+1} sentences. The sentences with the same number of unnegated predicate As are isomorphic: if we interchange the individual constants, the state description retains the same structure, namely the same number of unnegated Fs. Therefore, n+2 structure descriptions can be generated out of them in the language L(N+1). If the language has more predicates, each molecular predicate Q is a conjunction of the primitive predicates or their negation. In this case, a state description looks more complicated.

In the example above there are 4 structure-descriptions:
D1, D8, (D2∨D3∨D5), (D4∨D6∨D7)
In practice, this means that state descriptions that differ only in the names of individuals involved in them have the same structure. In words, these four structure descriptions state:

> 'Everything is F';
> 'Everything is –F'
> 'Two *F*s, one –F'
> 'One *F*, two –Fs'

Logical Width w of an expression M containing molecular predicates = the number of the Q-predicates of M.

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of L should differ only with respect to these two attributes. If a new predicate P_3, interpreted as Hard, were added, the c-values of hypotheses concerning individuals in L would change. Even if this simple scheme holds (or might hold) in a simple language, can it be adequate for the language of natural sciences? A similar requirement had been proposed by John Maynard Keynes, in the form of the Principle of Limited Variety. According to it, "...We seem to need some such assumption as that the amount of variety in the universe is limited in such a way that there is no one object so complex that its qualities fall into an infinite numbers of independent groups" (1921, 287).

Relative logical width q= the ratio of the logical width w over the number k of Q-predicates of the language L, i.e., q=w/k

6. **The c-function** is defined as follows: $c(h/e) = m(e\&h)/m(e)$.
The regular measure function *m* amounts to the unconditional prior probabilities of the hypothesis and the evidence. It fulfils two requirements:

(1) for any state description B in the language L_n, $m(B)>0$;
(2) the sum of the m-values of all the state descriptions in L_n is 1.

m(e) designates a measure assigned to the range of evidence e, i.e., the class of those state descriptions in which e holds.[2]
m(e&h) stands for a measure assigned to the intersection of the ranges R(h) and R(e), viz., R(h&e). In inductive logic, a part of R(e) is contained in R(h). The degree of confirmation function aims to calculate the degree to which R(e) is contained in R(h); in other words, it aims to calculate the degree that the evidence e (partially) entails the hypothesis h.[3]

The c-function satisfies the axioms of probability calculus
(1) L-equivalent pieces of evidences: If e and e' are L- equivalent, then c(h/e)=c(h/e').
(2) L-equivalent hypotheses: If h and h' are L- equivalent, then c(h/e)=c(h'/e).
(3) General Multiplication principle: $c(h_1\&h_2/e)=c(h_1/e) \cdot c(h_2/e\&h_1)$
(4) Special addition principle: If e, $h_1\&h_2$ are mutually exclusive, then $c(h_1 \vee h_2/e)= c(h_1/e) + c(h_2/e)$.

Since inductive logic deals with the relations between the ranges of sentences, it does not depend, according to Carnap, on the factual status of the sentences involved. It depends only on the *meaning* of the sentences involved, which is determined by the semantic rules of the language we use. This is supposed to make inductive logic purely analytic and internal to the language we use.
For Carnap, the sentence:

S: ‘The probability that it will rain tomorrow, given the evidence of the meteorological observations, is 1/5’

means that there is a logical connection between these two propositions and this logical connection has the value of 1/5 (LFP, 30).

Since this relation is *logical*, S is (analytically) true.

In his ‘Replies and Systematic Expositions’, Carnap distinguished between the *value* of c(h/e) which is dependent on experience (and, hence, it cannot be a logical relation) and the acceptability of the axioms which determine the value of c(h/e) which, he claimed, are independent of experience.

Later on, Carnap took it that inductive logic is concerned with the *rational degree of belief*. Its task is to lead us to degrees of belief, which we can be defended

[2] m(e) coincides with the so-called null confirmation of the sentence e, i.e., with the confirmation of e on the basis of a tautology. It is easy to see that if the null confirmation is $c_0(e/t)$, then it is equal to m(e). Hence, m(e) stands for the prior probability of e.
[3] In the case of the deductive logic, where e entails h, the range of e is entirely contained in the range of h. We then speak of a relation of total inclusion between ranges.

as rational. For this purpose, he defined the so-called *credence-function* (ILII, 260) to designate a system of degrees of beliefs for a given field of propositions. This function must satisfy the Ramsey-De Finetti theorem, i.e., to be coherent,[4] (see above) and furthermore the Shimony theorem, i.e., to be strictly coherent.[5] To this credence-function corresponds a quantitative *credibility-function* (pretty much like the c- function). As Carnap put it: 'Any logical function is an adequate c-function if and only if it is such that, if it were used as a Cred(ibility)-function, it would lead to rational decisions' (RAE, 971-972)). Its definition is this:

> 'The credibility of a proposition H with respect to another proposition A, for a person X, means the degree of belief that X would have in H if and when his total observational knowledge of the world was A' (ILII, 262).

This function expresses 'the underlying permanent disposition for forming and changing his beliefs under the influence of his observations' whereas the credence-function stands for the temporary beliefs at various times. The requirements for the credibility function form the basis of the axioms of the inductive logic. From the requirement of coherence, it follows that the logic associated with the credibility function must contain the axioms of probability calculus. From the requirement of strict coherence, it follows that c(h) >0. Carnap enunciated a further requirement, viz., the requirement of *symmetry* with respect to individuals. According to this, the credibility of a function should be invariant with respect to any exchange of individuals.[6] Hence, the axioms of inductive logic follow from the requirements for the rationality of the credibility function.

The various c-functions[7]
The c^+-function. A possible and plausible candidate for the c-function is the c^+. How it is constructed? According to Carnap, it might be taken to be plausible to assign equal prior probabilities to the set of state descriptions in a language L. Hence, if B is the set of them, and b one of them, $m^+(b)=1/@$, where @ is the number of them. c^+ is

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[4] Ramsey and de Finetti used the system of betting quotients to account for the degree of belief of an individual in a certain hypothesis. If the system of bets for and against a hypothesis is coherent (de Finetti) or consistent (Ramsey) i.e., if it satisfies the axioms of the calculus, then its impossible the bet to be unfair (i.e., that one of the bettors will lose whatever the outcome be) or a 'Dutch Book' to occur. (See Ramsey, 1926) and de Finetti, 1936).
[5] A strictly coherent function is one which fulfils the following regularity condition: it ascribes a non-zero probability in every non-contradictory (molecular) proposition H (ILII, 261-262). This requirement follows from the fact that if the prior probability of a hypothesis H is zero, any confirmatory instance (i.e., any finite observational evidence) does not raise it above zero.
[6] The principle of symmetry is a requirement already laid down in his LFP (see 483ff). An m-function is symmetrical if it has the same value for any of a set of isomorphic states descriptions. This requirement amounts to saying that the c-function must treat on a par all structure descriptions since the latter are made out of the disjunction of the isomorphic state descriptions. This requirement compels him to take a certain candidate for his c-function, the c*.
[7] The problem of finding a partition, in a language L, to which one can assign equal prior probabilities (based on the classical principle of indifference) has given rise to two possible candidates of the c-function. Keynes named the two possibilities 'ratio' and 'constitution' cases respectively. He advocated the 'constitution' case which corresponds to c^+, noting, to use Carnapian terminology, that the states descriptions are *not* further decomposable, and therefore satisfy his modified principle of indifference (1921 64-65). The 'ratio' case (i.e., statistical frequencies) is not valid because there is no symmetry of relevance in knowledge. That is to say, if we accept that all possible ratios (i.e., frequencies) are equally probable, we can see that our relevant knowledge differ with regard to ratio we adopt. (For an elucidating example see Keynes op.cit., 64)

defined as the c-function based upon m^+. But it can be easily shown that c^+ is entirely inadequate as a concept of degree of confirmation.

Proof

Let us consider a language L(n+1), with F the only primitive predicate. We want to calculate the degree of confirmation of the hypothesis that the n+1th individual will have the property F, (h says: 'Fa_{n+1}') given the evidence that all individuals examined so far had the property F (e says: $Fa_1+Fa_2+...+Fa_n=S$). The m^+ function assigns equal probabilities t_i the state descriptions of the individuals of the language. There are n+1 individuals and hence 2^{n+1} state descriptions.

$e\Leftrightarrow(S\&Fa_{n+1}) \vee (S\&-Fa_{n+1})$

So, $m^+(e)=1/2^{n+1}+ 1/2^{n+1}= 2/2^{n+1}$

Moreover, $m^+(e\&h)= 1/2^{n+1}$, since e&h describes a state description.

So, $c^+(h/e)= m^+(e\&h)/m^+(e) = (1/2^{n+1})/ 2/2^{n+1}=1/2=m^+(h)$

The evidence adds nothing to the probability of the hypothesis h. Its degree of confirmation c^+ remains the came, no matter whether one hundred or fifty or even zero observations of individuals being F have preceded. Therefore c^+ is a strongly counter- inductive function.

The c*-function. Carnap went on to attribute equal probabilities to the structure descriptions in a language L. If we translate this kind of equiprobability into simpler terminology, we can see that what becomes equiprobable is the relative frequency (or ratio) of the disjunctions with the same number of isomorphic state descriptions. That is, the structure description S with one unnegated predicate (in an one-predicate-language) has the same initial probability with the structure description S containing two unnegated predicates, and so on. Therefore the $m^*(S)=1/n+2$, in a language Ln+1. Then c* is defined in accordance with m*. This function amounts to Laplace's measure of probability or rule of succession.

Proof

Let's define a language L(n+1) with n+1 individuals and one predicate F, and form the hypothesis h that the individual n+1 has the property F (i.e., $h=Fa_{n+1}$). Evidence e says that *r* of the *n* individual examined, have the property F; i.e.,

$e\Leftrightarrow \pm Fa_1\& \pm Fa_2 \&...\& \pm Fa_n$

where r of them have the property F. We want the degree of confirmation c*(h/e). There are 2^{n+1} states descriptions of the individuals described in this language with respect to the two predicates F and - F. They obviously have the form

$Q_i(x)= \pm Fa_1\& \pm Fa_2\&...\&\pm Fa_{n+1}$

There are n+2 structure descriptions of the n+1 individuals. Let's call them Y. r is the number of unnegated Fs in each Y and it ranges from 0 to n+2. So Y=0 corresponds to the structure description with none unnegated F. Proceeding in the same way we see that Y=n+1 corresponds to the structure description with n+1 unnegated Fs.

m* assigns equal probabilities to each structure description. So m*(Y)=1/n+2.

If Y=r, there are ${}^{n+1}C_r$ possible ways of getting the number r of the unnegated F's in a disjunction of n+1 individuals. We assume that all these ways are equiprobable. So the ${}^{n+1}C_r$ ways are distributed in n+2 structure descriptions.

c*(h/e)=m*(e&h)/m*(e).

$e \Leftrightarrow (e\&Fa_{n+1}) \vee (e\&\text{-}Fa_{n+1})$
So, $m^*(e)=(1/n+2 \cdot 1/{}^{n+1}C_r) + (1/n+2 \cdot 1/{}^{n+1}C_r)$
h&e is a structure description distributed in ${}^{n+1}C_{r+1}$ possible ways.
So $m^*(e\&h)=(1/n+2) \cdot (1/{}^{n+1}C_{r+1})$

Therefore, c*(h/e)= r+1/n+2

This result amounts to the Laplace's rule of succession. If r=n, then c*(h/e)=1.
This rule is strongly inductive, since it tends to unity when r tends to n.
Its main justification comes from the assignment of equal probabilities structure descriptions. As we have just seen, a different application of the principle of indifference leads to a radically different—non-inductive—rule. Hence, the principle does not have the logical status that is needed for it to justify inductive logic.

Example

State Descriptions	weight	Structure Descriptions	weight	m^+	m*
D1. Fa_1 & Fa_2 & Fa_3	1/8	Everything is F	1/4	1/8	1/4
D2. Fa_1 & Fa_2 & -Fa_3	1/8			1/8	1/12
D3. Fa_1 & -Fa_2 & Fa_3	1/8	Two Fs, one not-Fs	1/4	1/8	1/12
D5. -Fa_1 & Fa_2 & Fa_3	1/8			1/8	1/12
D4. Fa_1 & -Fa_2 & -Fa_3	1/8			1/8	1/12
D6. -Fa_1 & Fa_2 & -Fa_3	1/8	One F, two not-Fs	1/4	1/8	1/12
D7. -Fa_1 & -Fa_2 & Fa_3	1/8			1/8	1/12
D8. -F a_1 & -Fa_2 & -Fa_3	1/8	Everything is not F	1/4	1/8	1/4

Let's first examine the c^+-function. Take the hypothesis h=Fa_3. Its range is {D1, D3, D5, D7}
$m^+(h)=1/2$
Let's suppose that the evidence is
e=Fa_1
$m^+(e) =1/2$

Then, $m^+(h\&e)=1/4$
Hence,

$c^+(h/e)= m^+(h\&e)/ m^+(e) = 1/4/1/2=1/2$
which is equal to $m^+(h)$—and hence, it does not confirm it.

Take now the c^*-function
$h=Fa_3$ {D1, D3, D5, D7}
$m^*(h)=1/2$
$e=Fa_1$
$m^*(e) =1/2$
$m^*(h\&e)=1/3$
Hence,
$c^*(h/e)= m^*(h\&e)/m^*(e) = 1/3/1/2=2/3$.
On this function, the evidence confirms the hypothesis.

7. Carnap takes it that the rule c* captures his view that the most important inductive inference is the predictive inference (LFP, 568). This rule might be interpreted as suggesting that the probability of a prediction that an unobserved individual a_{n+1} will have the property F is equal to the relative frequency r/n. To avoid this, Carnap generalises this rule as follows:

c*(h/e)=c(h/e')= r+w/n+k,

where r/n is the relative frequency of a predicate F in the observed sample and w/k is the logical width of F. Hence, c* becomes a semi-logical function. Before any observation is available, it yields an a priori probability equal to the logical width of F. As the evidence rolls in, the relative frequency (i.e., a factual component) gains more influence on the value of c*. The value of c* approaches the value of the observed relative frequency as the size of the sample increases.

In his (1952), Carnap tried to integrate all rules of inductive method, by introducing a new logical parameter λ. As noted already, the c*-function combines two factors: one empirical, and one logical. Carnap extends this to every possible c-function. He introduces a new λ-function, in which the parameter λ captures the relative weight given to the logical component of the rule.

Carnap first equates the c-function with a G-function, which depends on

a) the number k of the molecular predicates of the language in use;
b) the number n of the individuals of the sample; and
c) the number r of the confirming instances of a hypothesis.

So, c(h/e)=G(k, n, r)

Then, he makes G dependent on the two factors mentioned above, namely the relative frequency r/n and the logical width w/k of a predicate F. G takes values within the interval (r/n, w/k) (1952, 25). In the simple case of a language with one predicate F, w=1. Then G-function is constructed as the sum of the weighted mean of the two factors. Keeping, by convention the weight of r/n standardized and taking λ to be the weight of the logical factor 1/k, he constructs the following function:

$$G(k,r,n) = \frac{r + \lambda(k,r,n)/k}{n + \lambda(k,r,n)}$$

and solving for $\lambda(k,r,n)$, he gets:

$$\lambda(k,r,n) = \frac{n \cdot G(k,r,n) - r}{\frac{1}{k} - G(k,r,n)}$$

The advantage of such a function is that λ does not depend on n and r (op.cit, 29). Hence, instead of λ(k, r, n), he has λ(k).

The result of all this, is the following function:

$$c(h_i / e_i) = \frac{r + \lambda/k}{n + \lambda}$$

where h_i is the hypothesis that the next predictive instance of an individual not yet observed will have F.
With one further qualification, these two functions produce all possible inductive rules! The qualification is that λ can be either independent of k or dependent on it.

In the first case we have:
(a) If $\lambda = \infty$, we get the c^+-function.
$c(h_i/e_i)=1/k$

If we let K be any state description with n individuals, then $m_\infty(K)=1/k^n$. This function is counter-inductive, since no matter what factual information is obtained, its value does not change.

(b) If $\lambda=0$, then get the so-called straight rule of confirmation, viz., the relative frequency.
$c(h_i/e_i)=r/n$

The logical factor disappears and c simply takes the empirical value r/n.

(c) λ might be given different discrete values. For instance, if we set $\lambda=2$, we get the modified Laplace's rule of succession:

$$c(h_i / e_i) = \frac{r + \frac{2}{k}}{n + 2}$$

Carnap actually modifies this rule because it leads to inconsistencies. He restricts it to primitive properties only (which always have width w=k/2 and relative width w/k=1/2) (op.cit., 35)
Carnap tries more small numbers for λ, and gives a table for the corresponding numerical values of c. (op.cit., 37)

In the case in which λ depends on k, Carnap gets his own c*-function. λ^* depends on k (i.e., $\lambda^*(k)=k$) and therefore,

$$c^*(h_i/e_i) = \frac{r+1}{n+k}$$

Carnap then generates a series of c*- functions: λ (k)=Ck, where C is a constant coefficient (C=1,2,3...).

The result of all this is the following. In the first case, where λ is independent of k, the smaller the value of λ, the sooner we learn from experience. But, what we can get at best is Laplace's rule of succession. Moreover, since λ does not change with k, any change in the language does not affect the values of c. In the second case, where λ is dependent on k, Carnap's favourite function c* changew in accordance with k, i.e., in accordance with the language (op.cit., 49).
Hence, there is a multiplicity of inductive rules. Some are (more or less) adequate, others are inadequate (c+, or c* with C>2). It is up to the scientists to make up their minds and to choose among them the one that they feel are the more appropriate for their purposes. They can change them as they change their automobiles (op.cit., 55)! To paraphrase Chairman Mao's dictum, Carnap seems to say 'Let a hundred inductive methods bloom'.

In any case, Howson and Urbach state, the idea of adjusting the value of λ appropriately, 'calls into question the fundamental role assigned to his systems of inductive logic by Carnap. If their adequacy is itself to be decided empirically, then the validity of whatever criterion we use to assess that adequacy is in need of justification, not something to be accepted uncritically' (1989, 55).

Subjective Bayesianism

Also known as subjective **Bayesianism**, this conception takes probabilities to be *subjective degrees of belief*. It opposes the objective or logical interpretation of probability in denying that there is such thing as rational degree of belief in the truth of a proposition. Each individual is taken to (or allowed to) have her or own subjective degree of belief in the truth of a certain proposition. Given that the probability calculus does not establish any probability values, subjectivists argue that it is up to the agent to supply the probabilities. Then, the probability calculus, and Bayes's theorem in particular, is used to compute values of other probabilities based on the prior probability distribution that the agent has chosen. The only requirement imposed on a set of degrees of beliefs is that they are probabilistically coherent, that is that they satisfy the axioms of the calculus. The rationale for this claim is the so-called Dutch-book theorem.

Bayes's Theorem is a famous theorem of the probability calculus (see above). Let *H* be a hypothesis and *e* the evidence. Bayes's theorem states: P(H/e)=P(e/H)P(H)/P(e), where P(e)=P(e/H)P(H)+P(e/-H)P(-H).
The unconditional prob(H) is called the prior probability of the hypothesis;
the conditional P(H/e) is called the posterior probability of the hypothesis *given* the evidence;
the P(e/H) is called the *likelihood* of the evidence given the hypothesis.
Note that if e refutes H, then P(e/H)=0;
If H entails e, then P(e/H)=1

(The Theorem can be easily reformulated so that background knowledge is taken into account.)

Dutch-book theorem. This is a significant mathematical result—proved by Frank Ramsey and, independently, by Bruno de Finnetti—which is meant to justify the claims that subjective degrees of beliefs (expressed as fair betting quotients) satisfy Kolmogorov's axioms for probability functions. The key idea is that unless the degrees of beliefs that an agent possesses, *at any given time*, satisfy the axioms of the probability calculus, she is subject to a Dutch-book, that is, to a set of synchronic bets such that they are all fair by her own lights, and yet, taken together, make her suffer a net loss come what may. The monetary aspect of the standard renditions of the Dutch-book theorem is just a dramatic device. The thrust of the Dutch-book theorem is that there is a *structural incoherence* in a system of degrees of belief that violate the axioms of the probability calculus. The demand for probabilistic coherence among one's degrees of belief is a *logical* demand: in effect, a demand for logical *consistency*.

Conditionalisation

There are have been attempts to extend Bayesianism and the requirement of coherence to belief updating. The key thought here is that should update their degrees of beliefs by *conditionalising* on the evidence. Accordingly, the transition from the old probability $Prob_{old}(H)$ that a hypothesis H has had to a new one $Prob_{new}(H)$ in the light of incoming evidence e, is governed by the rule: $Prob_{new}(H)=Prob_{old}(H/e)$, where *e* is the total evidence, and $Prob_{old}(H/e)$ is given by Bayes's Theorem. There are two forms of conditionalisation: strict conditoinalisation, where the probability of the learned evidence is unity; and Jeffrey-conditionalisation, where only logical truths get probability equal to unity. Conditionalising on the evidence is purely *logical* updating of degrees of belief. It's not ampliative. It does not introduce new content; nor does it modify the old one. It just assigns a new probability to an old opinion.
The justification for the requirement of conditionalisation is supposed to be a diachronic version of the Dutch-book theorem. It is supposed to be a canon of rationality (certainly a necessary condition for it) that agents should update their degrees of belief by conditionalising on evidence. The penalty for not doing this is liability to a *Dutch-book strategy*: the agent can be offered a set of bets *over time* such that a) each of them taken individually will seem fair to her at the time when it is offered; but b) taken collectively, they lead her to suffer a net loss, come what may. In this context, induction, then, is the process of updating already given degrees of belief and its justification gives way to the problem of justifying conditionalisation on the evidence.

As is generally recognised, the penalty is there on a certain condition, viz., that the agent *announces in advance* the method by which she changes her degrees of belief, when new evidence rolls in, *and* that this method is different from conditionalisation. Critics of diachronic Bayesianism point out that there is no general proof of the conditionalisation rule. In fact, there are circumstances under which conditionalisation is an *inconsistent* strategy. When an agent is in a situation in which she contemplates about her $Prob_{new}(\text{--})$, she is in a *new* and different (betting) situation in which the previous constraints of $Prob_{old}$ need not apply. A case like this is when the learning of the evidence *e* does upset the conditional probability Prob(--/e).

Bayesian theory of Confirmation
According to Bayesian theories of confirmation
a) confirmation is a relation of positive relevance, viz., that a piece of evidence confirms a theory if it increases its probability;
e confirms h iff $P(h/e)>p(h)$
b) this relation of confirmation is captured by Bayes's theorem;
c) the only factors relevant to confirmation of a theory are its prior probability, the likelihood of the evidence given the theory; and the probability of the evidence;
d) the specification of the prior probability of (*aka* prior degree of belief in) a hypothesis is a purely subjective matter;
e) the only (logical-rational) constraint on an assignment of prior probabilities to several hypotheses should be that they obey the axioms of the probability calculus;
f) hence, the reasonableness of a belief does not depend on its content; nor, ultimately, on whether the belief is made reasonable by the evidence;
and g) degrees of belief are probabilities and that belief is always a matter of degree.

There is still a pervasive dissatisfaction with subjective Bayesianism. This dissatisfaction concerns all of the theses (a) to (f) above, but it is centred mostly around the point that subjective Bayesianism is *too* subjective to offer an adequate theory of confirmation and of rational belief. It is typically claimed that purely subjective prior probabilities fail to capture the all-important notion of rational or reasonable degrees of belief.

Bayesians reply by appealing to the **convergence of opinion theorem**, which shows that the actual values assigned to prior probabilities do not matter much since they 'wash out' in the long run; that is, they converge to the same value. Suppose, for instance, that a number of individuals assign different subjective prior probabilities to some hypothesis *h*. Suppose further that a sequence of experiments is performed and the individuals update their prior probabilities by means of conditionalisation. It can be proved that after a point, their posterior probabilities converge to the same value. This result is supposed to mitigate the excessive subjectivity of Bayesianism.

Successes of the Bayesian theory
The ravens paradox. This is a paradox of confirmation, first noted by Carl Hempel. It concerns the confirmation of universal generalisations, but it took its name from the example that Hempel used to illustrate it, viz., *all ravens are black*. There are three intuitively compelling principles of confirmation which cannot be jointly satisfied; hence the paradox.
First, Nicod's principle (named after the French philosopher Jean Nicod (1893-1924)): a universal generalisation is confirmed by its positive instances. So, that all ravens are black is confirmed by the observation of black ravens.
Second, the principle of equivalence: if a piece of evidence confirms a hypothesis, it also confirms its logically equivalent hypotheses.
Third, the Principle of relevant empirical investigation: hypotheses are confirmed by investigating empirically what they assert.
Take the hypothesis (H): All ravens are black. The hypothesis (H') *All non-black things are non-ravens* is logically equivalent to (H). A positive instance of H' is a white piece of chalk. Hence, by Nicod's condition, the observation of the white piece of chalk confirms H'. Hence, by the *principle of equivalence*, it also confirms H, that is that all ravens are black. But then the principle of relevant empirical investigation is violated. For, the hypothesis that all ravens are black is confirmed not by examining

the colour of ravens (or of any other birds) but by examining seemingly irrelevant objects (like pieces of chalk or red roses). So at least one of these three principles should be abandoned, if the paradox is to be avoided.

The Bayesian solution:

H: All Ravens are Black—All Rs are Bs

H': All Non-black things are non-ravens—All -Bs are -Rs

Positive instances of H: Ra&Ba

Positive instances of H': -Ba&-Ra

Evidence e: Ra&Ba

Evidence e': -Ba&-Ra

P(H/e)=P(e/H)P(H)/P(e)

P(H/e')=P(e'/H)P(H)/P(e')

P(e/H)=P(e'/H)=1

Hence

P(H/e)/P(H/e')=P(e')/P(e)

But P(e')>>P(e) because there are very many more things which are non-Black &non-Ravens than Black Ravens. Hence P(e')/P(e)>1.

Hence, P(H/e) >> p(H/e').

Hence e confirms H a lot more than e' confirms H'.

Surprising predictions

Predictions whose probability is low given background knowledge confirm more than already known evidence.

The Bayesian account:

P(H/e)=P(e/H)P(H)/P(e)

Suppose that P(e) is low and that P(e/H)=1 (H entails e). Then

P(H/e)= P(H)/P(e)

The lower the P(e) the higher the P(H/e).

A problem for the Bayesian theory of confirmation, first identified by Glymour, is known as the **old evidence problem.** Suppose that a piece of evidence *e* is already known (it is, that is, an old piece of evidence relative to the hypothesis *H* under test). Its probability, then, is equal to unity (prob(e)=1). Given Bayes's theorem, it turns out that this piece of evidence does not affect at all the posterior probability (prob(H/e)) of the hypothesis given the evidence: the posterior probability is equal to the prior probability, i.e., prob(H/e)=prob(H). This, it has been argued, is clearly wrong, since scientists typically use known evidence to support their theories. Therefore, there must be something wrong with Bayesian confirmation. Bayesians have replied by adopting a counterfactual account of the relation between theory and old evidence. They argue as follows. Suppose that *B* is the relevant background knowledge and *e* is an old (known) piece of evidence—that is, *e* is actually part of *B*. In considering what kind of support *e* confers on a hypothesis *H*, we subtract counterfactually the known evidence *e* from the background knowledge *B*. We therefore presume that *e* is not known and ask: what would the probability of *e* given *B*-*e*? This will be less than one; hence, the evidence *e* can affect (that is, raise or lower) the posterior probability of the hypothesis.