ANTI-NOMINALISTIC SCIENTIFIC REALISM: A DEFENCE

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1 Introduction

Philosophy of science proper has been a battleground in which a key battle in the philosophy of mathematics is fought. On the one hand, indispensability arguments capitalise on the strengths of scientific realism, and in particular of the no-miracles argument (NMA), in order to suggest that a) the reality of mathematical entities (in their full abstractness) follows from the truth of (literally understood) scientific theories; and b) there are good reasons to take certain theories to be true.[1]

On the other hand, arguments from the causal inertness of abstract entities capitalise on the strengths of scientific realism, and in particular of NMA, in order to suggest that a) if mathematical entities are admitted, the force of NMA as an argument for the truth of scientific theories is undercut; and b) the best bet for scientific realism is become Nominalistic Scientific Realism (NSR) and to retreat to the nominalistic adequacy of theories.

In what follows, I will try to show that anti-nominalistic scientific realism is still defensible and that the best arguments for NSR fail on many counts. In Section 2, I will argue that there are good reasons not to read NMA as being at odds with the reality of abstract entities. In Section 3, I will discuss what is required for NSR to get off the ground. In Section 4, I will question the idea of the nominalistic content of theories as well as the idea of causal activity as a necessary condition for commitment to the reality of an entity. In Section 5, I will challenge the notion of nominalistic adequacy of theories. In Section 6, I will try to motivate the thought that there are mixed physico-mathematical truth-makers, some of which are bottom-level. Finally, in Section 7, I will offer a diagnosis as to what the root problem is in the debate

[1] The best defence of this argument is by Colyvan (2003).

between Platonistic Scientific Realism and NSR and a conjecture as to how it might be (re)solved.

2 From Scientific Realism to Platonism

A central argument for the reality of mathematical entities comes straight from the philosophy of science. The indispensability argument (*IND-A)*, put in a nutshell, suggests that the existence of abstract entities follows from the truth of scientific theories. There has been a lot of discussion about the exact formulation of *IND-A*, but I take it that the simplest and most explicit way to think of it is this:

> *IND-A*
>
> 1. If they are true, scientific theories (taken literally), imply the existence of abstract entities (numbers, sets etc.).[2]
> 2. Scientific theories are true (weaker: there are good reasons to believe that scientific theories are true).
> *Therefore*
> There are abstract entities (weaker: there are good reasons to believe that there are abstract entities).

Seen that way, the claim of indispensability does not come into the argument directly, but is part of its defence. 'Indispensability' supplements the Quinean criterion of ontic commitment. The latter, as is well known, has to do with the values of the variables in the canonical notation of quantification of a theory. Roughly put, a theory is ontically committed to whatever its variables of quantification range over. But then to go from ontic commitment to what there is, *truth* is required. What a theory is committed to is one thing, what there is, another.[3] The two are linked by the truth of the theory. But before any ontic commitments are being read off the theory, the theory should be taken at face-value. More importantly, if the entities which a face-value reading implies are dispensable, that is, if the theory can be re-written (paraphrased, as Quine would put it) without implying commitment to such entities and without losing in explanatory power etc., the question of commitment becomes moot. So: part of the defence of *IND-A* is that a) quantifying over (abstract) mathematical objects is indispensable in science; and b) theories which quantify over mathematical entities (as well as physical entities—like electrons etc.) are highly confirmed; hence (likely to be) true.

What needs to be stressed (for the purposes of the issue at hand) is the general idea behind *IND-A*, viz., that scientific realism implies Platonism—or at least that it contradicts nominalism.[4] Concomitant to this general idea is an epistemological

[2]This, of course, assumes standard semantics for mathematics, according to which mathematical theories—literally understood—refer to abstract entities.

[3]Here is what Quine (1980: 15) says: "We look to bound variables in connection with ontology not in order to know what there is, but in order to know what a given remark or doctrine, our or someone else's, says there is".

[4]The stronger claim, strictly speaking, follows only if there is further argument to the effect that abstract entities should not be understood as mental constructions. But I take this for granted in this paper. For some interesting discussion of reductionist versions of nominalism, see Heck (2000).

bonus. If the truth of scientific theories can be known, there can be knowledge of abstract entities too. Perhaps, there can be knowledge of mathematical truths, too—though not necessarily of truths of *pure* mathematics.

The epistemic optimism characteristic of scientific realism is based on NMA. The argument, roughly put, is that empirical success (suitably regimented so as to include novel predictions and the like) offers good reasons to believe in the truth of theories, since it is best explained by the claim that theories are true. Thus conceived, NMA is blind to a distinction between abstract entities and concrete ones insofar as commitment to both types is implied by the truth of (literally understood) scientific theories. In all its generality, NMA does not demand a *causal* explanation of the empirical success of theories. In particular, it does not demand that the entities that are required for the explanation of certain empirical phenomena should be concrete physical entities. This is as it should be, for at least two reasons that are rarely noted. *First*, any serious (let alone the best) explanation of at least some empirical/predictive successes will have to cite/employ natural laws, and these are in no sense concrete entities. Nor are laws causal entities. It might well be that *c* causes *e* via a law (i.e., *c* is nomologically sufficient for *e*), but laws themselves do not cause anything, though they are explanatory of what happens in the world. (Newton's law of gravity does not cause things to fall to the ground, although it governs—arguably—their relevant behaviour; nor does Newton's laws cause Kepler's laws and the like.)[5] *Second*, any serious (let alone the best) explanation of at least some empirical/predictive successes will cite/employ *models*. These are not concrete causal entities, and yet they play an important explanatory role. Hence, it is a good thing that NMA is not tied exclusively to causal explanations of the successes of theories.

In its usual formulations, the indispensability argument is cast in terms of confirmation of scientific theories and a standard worry (dressed up as an objection) is that confirmation cannot reach the abstract objects posited by theories and the claims made about them. This objection is flatly question-begging. It presupposes that the relation of confirmation mirrors (or ought to mirror) causal relations; that only whatever is concrete and causal is such that assertions about it can enter into relations of confirmation with observational reports. If this presupposition were to be granted, universal law-like statements would not be confirmable, since if true, they are made true by entities which are not concrete and causal. Nor would any hypothesis involving theoretical models ever be confirmable.

If *IND-A* is sound, there are mixed facts, viz., structured entities, whose structure is made up by concrete *and* abstract elements. More specifically, some mixed statements implied by scientific theories (of the form 'P is (or has the property) M', where 'P' is meant to refer to a physical object and 'M' to a mathematical one) are made true by mixed facts: mathematical entities (and properties) are somehow related to physical entities to render mixed statements true. *IND-A* need not imply that there is causal contact (or causal glue) between physical entities and mathematical entities that make up the mixed fact.[6] What *IND-A* does imply is that mathematical entities

[5]Even so-called causal laws (e.g., smoking causes lung-cancer) are not causal entities (they have no causal powers themselves), though they capture, arguably, causal relations among properties.

[6]This is not particularly surprising. If we take facts seriously as structured entities, (where a property is attributed to an object or a relation is said to hold between a number of objects) the relation that 'holds

are required for the truth of scientific theory. This should not be confused with the stronger claim that they are *causally* required for their truth. (We shall return to this issue in Section 6).

3 From Scientific Realism to Fictionalism

Resistance to Platonism has also found its springboard in the philosophy of science. Hartry Field's (Field 1980) argument against Platonism has been based on the claim that mathematics is dispensable in science, viz., that scientific theories need not employ vocabulary that purports to refer to mathematical objects; hence, the truth of (suitably reformulated) scientific theories does not imply the reality of abstract entities. This is not an argument *for* fictionalism, viz., the view that there are no mathematical objects, but it paves the way for it. What is normally added to get fictionalism is the general idea that there is something *deeply* suspect with alleged abstract entities. That is, the very idea of abstractness eradicates any ontological pedigree. Concomitant to this general idea is an epistemological liability, viz., that precisely because of their abstractness, alleged abstract entities would be unknowable.

A standard argument against Field's anti-platonist move is that scientific theories (especially high-level ones) resist nominalization: they resist a nominalism-friendly reformulation that implies commitment only to concrete entities. This is an empirical matter in that one has to sit down and sketch, at least, how a nominalism-friendly reformulation of *each and every* scientific theory can be achieved. There is no reason to pause over it now, since there is a general strategy for bringing together scientific realism and mathematical fictionalism, akin to the one Bas van Fraassen (1980) has pursued in the scientific realism debate. This is to introduce the concept of nominalistic adequacy and to argue that even if scientific theories cannot be nominalised, even if mathematics is theoretically indispensable, there is a way to avoid commitment to mathematical realism. It is enough, it is argued, for the applicability of scientific theories and for the explanation of their empirical successes that they are nominalistically adequate, where a theory T is nominalistically adequate if [and only if] "it is correct in its nominalistically-stated consequences (i.e., if it is correct in those of its consequences that do not quantify over mathematical entities" Leng 2005: 77).

Though he does not endorse mathematical fictionalism, Gideon Rosen (2001: 75) has characterised modally n-adequacy thus: A (mathematised) theory S "is nominalistically adequate iff the concrete core of the actual world is an exact intrinsic duplicate of the concrete core of some world at which S is true—that is, just in case things are in all concrete respects *as if* S were true". A concrete core of a possible world W is "the largest wholly concrete part of W: the aggregate of all concrete things that exist in W".

together' the elements that make up a fact is not causal; sometimes it is said to be formal (e.g., by E. J. Lowe) precisely in order to make this point clear. In a (different but related) sense, *any* fact is a mixed entity (because it comprises particulars and universals); and what binds all these constituents into a single entity is *not* causation.

We shall discuss this concept in some detail in Section 5, but for the time being let us focus on the general idea behind it, viz., that a theory can be *false* (if literally understood), and yet get everything right vis-a-vis whatever is concrete. We tend to forget that the consensus over the claim that scientific theories should be taken at face-value (they should be understood literally) is a hard-won one, fought against reductive strands in empiricism and syntactic instrumentalism. Literally understood, scientific theories purport to refer not just to unobervable entities (over and above observable ones), but also to mathematical entities too. In fact, as noted already, theories typically comprise a host of *mixed* statements, not just connecting the theoretical and the observational vocabulary but also connecting both vocabularies with a mathematical vocabulary. Mixed statements, under the assumption of a literal understanding of them, require mixed truth-makers or mixed facts. If, as it happens, some part of the required truth-maker is missing (as will be the case if there are no mathematical objects), two options are available to a would-be scientific realist. One is to go for a non-literal understanding of the mixed statements, thereby claiming that the *appropriate* truth-maker in not really mixed and hence that no part of it is *really* missing. The other option is to insist on a literal understanding of theories and to concede that they are false. Leaving the first option to the one side, the second option would be *prima facie* disastrous for realism, at least as an epistemic thesis. How, for instance, can this systematic and symptomatic falsity of theories explain their empirical successes?

There is a nominalism-friendly way out of this problem, but requires that there is a way to:

> a) carve up entities into two disjoint sets—the concrete and the abstract;
> b) disentangle whatever the theory asserts about concrete causal entities (its nominalistic content) from whatever it asserts about abstract ones; and
> c) show that whatever credit accrues to the theory from its applications to the world comes exclusively from its nominalistic content.

Assuming that (a) can be dealt with, what Mark Balaguer (1998: 130) has called 'nominalistic scientific realism' (NSR) aims mainly to deal with (b) above. He enunciates the following two theses:

> (NC) Empirical science has a purely nominalistic content that captures its 'complete picture' of the physical world.
> (COH) It is coherent and sensible to maintain that the nominalistic content of empirical science is true and the platonistic content of empirical science is fictional.

(NC) asserts precisely what needs to be shown. But the argument for it is just the fact that mathematical entities, if they exist at all, are causally inert (cf. 1998, 132). Hence, there would be no *causal* difference in the world, if they did not exist. Hence, there is a way the world is—causally—which is independent of any mathematical objects, which are causally inert anyway and hence cannot contribute to the way the world is causally. The nominalistic content (n-content) of the theory, then, is what the theory says about whatever is part of the causal structure of the world. (COH) follows rightaway and makes possible the claim that though, literally understood, scientific theories are false, it is enough for scientific realism to get right the nominalistic content of theories, since we do not lose "any important part of out picture of the physical world" (1998: 134). As Balaguer put it, "The nominalistic content of a theory T is just that the physical world holds up its end of the 'T bargain', that is, does its part in making T true" (1998:135). In the end, mathematical fictionalists do not have to replace platonistic scientific theories with nominalistic ones. They just need to argue that when these theories are accepted, we are committed only to the truth of their nominalistic consequences and not to the truth of their platonistic consequences. In other words, scientific realism implies commitment to the nominalistic adequacy of theories.

Note that the move from 'no causal difference' to 'no difference', which is required for the assumption that the causal image of the world is the *complete* image of the world, is fallacious. It would imply that laws of nature make no difference since they make no causal difference. But laws do make a difference, even if it is not causal. They are unifiers; or they govern/explain their instances; or (more importantly), being patterns under which sequences of events are subsumed, they *constitute* what we call the causal structure of the world, viz., they make causal happenings possible—where this relation of constitution is not, of course, causal. Laws, to repeat, do not cause anything to happen; laws just *are* and things happen in conformity to them.

This is not yet an objection against NSR, though it will be extended to one in the next section. For the time being, let us note that even if Balaguer's strategy were impeccable, there would still be need for an argument for part (c) of the tripartite strategy for NSR noted above. A form of this has come from Leng (2005). Her view is that the best bet for scientific realism is to go for NSR—the (alleged) platonistic content of theories is an extra burden that scientific realists cannot discharge. Her argument is this. If true, theories (literally understood) imply the existence of both unobservable *and* mathematical entities. But the mathematical entities are causally inert; hence, they won't be involved in any causal explanation of certain empirical successes of theories (e.g., a novel prediction). Hence, the no-miracle argument that scientific realists employ to ground their epistemic optimism no longer offers reasons to believe in the full truth (nominalistic + platonistic) of scientific theories. And yet, it could give us reason to believe that successful theories are nominalistically adequate. *Ergo*, scientific realism is in much better shape if truth is replaced by nominalistic adequacy.

4 On the Nominalistic Content of Theories

Drawing a sharp distinction between the concrete and the abstract is notoriously difficult, even if there are paradigmatic cases of entities that are concrete and entities that are abstract (see, for instance Dummett 1991: 239). Most typical criteria for this distinction (lack of spatio-temporal location and causal inertness) admit of interesting (though occasionally contentious) counterexamples. In any case, there is little doubt that mathematical objects count as abstract, if only because they are the paradigmatic cases of causal inertness.[7]

Be that as it may, there is a whole category of abstract objects whose existence is contingent (i.e., they do not exist necessarily) and also contingent upon the existence and behaviour of concrete objects. Examples of such objects are the Equator, the centre of mass of the solar system, or even thoughts (if dualism were right).[8]More importantly, abstract objects are the truth-makers of the descriptions of (most) theoretical models employed by theories. The Linear Harmonic Oscillator (LHO), for example, or the two-body Newtonian system, or a frictionless inclined plane are pertinent examples. It's tempting to conflate models with their descriptions. But if care is taken to draw this distinction, models are abstract objects that satisfy certain descriptions. They are not *pure* abstract entities since physical properties are ascribed to them, but they are abstract nonetheless—and certainly not causally efficacious. We can borrow Dummett's expression and call models 'physical abstract entities'.[9] Similarly, objects such as the Equator or the first Meridian are geometrical abstract object.

We may draw a distinction between Non-Mathematical Abstract Objects (NMAOs) and Mathematical Abstract (MAOs). Literally understood, theories imply commitment to a host of NMAOs; that is, to a host of causally inert entities. It is absurd to say that all these NMAOs are not explanatorily relevant to the successes of theories; nor that they contribute nothing to the explanation of concrete physical objects and their behaviour. A LHO, for instance, does explain why the period of a concrete pendulum is proportional to the square root of its length; it supports certain counterfactuals (e.g., about changes of the length of the pendulum); it unifies under a type a variety of resembling concrete objects. It follows that causal inefficacy is no reason to deny that some entity is part of reality. Causal inertia does not imply explanatory inertia.

A *prima facie* plausible riposte available to NSR is that all these entities are dispensable. But this reply would be too quick. Let us distinguish between two types of NSR: Lenient NSR and Austere NSR. The lenient version is tough on mathematical objects, but does allow non-mathematical abstract entities. The austere version puts a ban on anything abstract. The austere version should aim to dispense with all putative abstract objects by reformulating scientific theories so that they do away with them. I do not know whether this is feasible, but suppose it is. The result of this Herculean operation will be, in all probability, a massively complicated theory which would be unable to make any general claims about concrete objects. Generality requires abstractness: otherwise the general cannot cover the particular. There is not a scientific theory of concrete springs, and another of concrete pendula and

[7]Note that *IND-A* is an a posteriori argument for the existence of abstract mathematical objects.

[8]Since this paper was drafted, an important book has appeared by Wetzel (2009), in which she thoroughly defends the idea that there are plenty of non-mathematical abstract objects (notably: semantic types, but also structural types, like flags, sonatas and molecules), which are explanatorily indispensable and not amenable to nominalistic paraphrase.

[9]It bears stressing that the qualification 'physical' is meant to imply that some abstract objects are described by physical (as opposed to mathematical) predicates. Types (e.g., TIGER or GRIZZLY BEAR, or Beethoven's FIFTH SYMPHONY) are abstract object of this sort (cf. also Wetzel 2009).

another of . . . : there is a theory of the linear harmonic oscillator, which covers many concrete structures that are inexact tokens of the linear harmonic oscillator.

The lenient version of NSR has, at least, the resources for the development of simple, explanatory and unified scientific theories via representational devices that employ NMAOs. The latter, among other things, provide the resources for the formulation of comprehensive laws. If NMAOs are (allowed to be) part of the content of scientific theories, the very idea of a sharply delineated nominalistic content of a theory that bans abstract entities altogether becomes otiose. For NMAOs (including laws) play a key role in specifying what the theory asserts about concrete objects and their behaviour. They also play a key role in explaining the behaviour of concrete objects. What is more, part of the identity of some NMAOs (more particularly, of models) are mathematical entities, like phase spaces, vector spaces and groups (cf. French 1999). And since NMAOs are, after all, abstract entities, there is no principled problem in having *mathematical* abstract objects as part of their constitution. So if NMAOS are explanatory, so are those mathematical entities that are part of their constitution.

Friends of lenient NSR might claim that the employment of descriptions that, taken at face value, refer to alleged abstract objects are purely descriptive and representational devices which, though expressionally and theoretically indispensable, are *not* metaphysically indispensable. Here again, however, the only general (and initially plausible) argument for the alleged metaphysical dispensability of abstract objects comes from their causal inertness, and this is not enough to deny existence. Abstract entities can still be explanatorily indispensable and explanatorily efficacious as well.

Pincock (2007) has pressed a point like this, but perhaps not far enough. His idea is that there are explanations and explanatory patterns (which he calls abstract or structural) which are part and parcel of the content of scientific theories. He restricts his attention to explanations that involve mathematical abstract objects (though it is clear he would not object to generalising them to NMAOs). These explanations proceed on the basis of descriptions of a physical system at a higher level of generality than its concrete physical constitution, by ignoring the microphysical properties of the system under study. (I would add: by ignoring the actual physical realisations of a NMAO, which in many cases might be inexact and approximate; that is, by replacing concrete physical systems by abstract physical systems, which are modelled/represented by a mathematical structure). Pincock argues that this kind of explanation is important for the generality of explanations offered by physical theory. As noted above, the generality of physical explanation (and its applicability) requires abstract physical entities.

The conclusion Pincock draws is that mathematics is epistemically indispensable to science (because we are often ignorant of the detailed physical or microphysical facts that might realise a certain abstract explanatory pattern) and claims (2007: 263–4) that epistemic indispensability is compatible with metaphysical dispensability. To be sure, he goes on to qualify this point by noting that the metaphysical dispensability of mathematical objects cannot be warrantedly asserted because, strictly speaking, the nominalistic content of theories is indeterminate: "our theories, understood in the light of the evidence we have, do not determine a collection of physical claims that we could view as the nominalistic content of these theories" (2007: 267). I do not doubt that Pincock is right, but his point is epistemic and as such it can only show that we have no good reasons, given the evidence, to take it for granted that mathematical entities are metaphysically dispensable. If what said above is broadly correct, there is a stronger position to occupy, viz., the very idea of an abstract-entities-free nominalistic content of theories is hollow. Abstract entities (both non-mathematical and mathematical) get entry visas because very little interesting (general and explanatory) can be said about the physical world without being committed to them.

So far, I have claimed that the fact that abstract objects are causally inert is not a reason to make us suspicious about their ontic status. Still, the question remains: if they are not given to us causally, how are they given to us? The answer to this question is: via a (suitably generalised) Fregean context principle (CP). Briefly put, the idea is that terms that purport to refer to abstract objects have their reference fixed by the contribution these terms make to what is required to determine the truth of the sentences in which they occur. Reference, in other words, is fixed via the truth of sentences in which certain terms occur. Actually, CP is the only non-question-begging way to settle the issue of the reference of terms that purport to refer to abstract objects; hence, the only way to have access to abstract entities. Abstract objects can neither be encountered in experience nor be presented in it. They cannot be initiators, or links in, causal chains that end up in perceptual states. To think otherwise is to have the wrong idea of what abstract objects should be; it is to view abstract objects as actual physical objects stripped of their causal powers.[10] But abstract objects constitute a different kind of object. It is not surprising then that their mode of knowledge is difference.[11]

Note that CP meshes very well with *IND-A*. The punch-line of the latter is that literal understanding fixes what conditions must be fulfilled for a theory to be true and truth determines that these conditions are satisfied: literal understanding + truth fix what is real. On this account, the real can be either concrete or abstract and there is no non-question begging further criterion to fix what should count as real. Unless explanation is conflated with causal explanation, abstract objects can be explanatorily relevant (say, by promoting unification), while not being causally relevant. And unless evidence is taken to be direct sensory evidence, there can be evidence for the reality of abstract objects via the evidence for the truth of theories in which they are being ineliminably referred.

[10] Being causally active, that is having causal powers, is a criterion of objecthood. This is sometimes called the Eleatic Principle. According to the Eleatic Stranger in Plato's *Sophist* (247 D-E): . . . everything which possesses any power of any kind, either to produce a change in anything or to be affected even in the least degree by the slightest cause, though it be only on one occasion, has real existence". Graham Oddie (1982) has forcefully criticised this principle. In any case, it is not the only criterion of objecthood. Actually, making it a *sine qua non* condition for objecthood begs the question.

[11] For some stimulating discussion of the Fregean Context Principle in relation to nominalism, see Heck (2000).

5 On Nominalistic Adequacy

Let us *presume* we can make good sense of the idea of the nominalistic content of a theory and pay some attention to the concomitant idea of nominalistic adequacy. The significance of this idea is that a theory can be nominalistically adequate and yet false (in that there are no mathematical entities). It is further argued that a nominalistically adequate theory (which is not just an empirically adequate theory) is exactly as explanatory of the observable phenomena as a platonistically adequate theory that assumes the existence of abstract entities—since the latter make no contribution to the causal explanation of the observable.

There is first an issue with the very idea of characterising n-adequacy. Leng's characterisation, stated as it is in terms of the truth of nominalistically-stated consequences of a theory is problematic.[12] As Jeff Ketland has noted (private communication), the required notion of n-adequacy should be model-theoretic. [13]

Briefly put, a theory T is n-adequate if a sub-structure of a model of the theory (this substructure which is fit for the representation of nominalistic facts) is isomorphic to the causal structure of the world. But now we have quantified over models—that is mathematical objects. Even if this objection is not fatal for the use of the concept of n-adequacy by the advocate of NSR, it would surely remove a lot of the attraction of NSR. Their advocates would have to have a fictionalist stance towards a central building block of their own account of how theories latch onto the world.

NSR forfeits the idea of a mathematics-free reformulation of scientific theories. This kind of situation leads to an interesting case of underdetermination, whereby the nominalistic content of a theory underdetermines its *full* content. We can eas-

[12]Even if a syntactic characterisation of n-adequacy were adequate, the following would be a problem. Theories yield consequences only with the aid of auxiliary assumptions. The claim then of n-adequacy would have to be that a theory T is n-adequate if for *all* auxiliaries M cast in mathematical language, M&T yield no extra nominalist consequences that do not follow from T alone. If this were *not* the case, some of the n-content of T would depend on the truth of mathematical claims. The only way to secure this does not happen is to retreat to the conservativeness of mathematics.

[13]According to Ketland (private communication), to get to a characterisation of n-adequacy, we need:

> a) a 2-sorted theory-formulation language L, with two variable sorts (one ranging over concreta and the other ranging over abstracta), and three kinds of predicate: primary, mixed and secondary; and
> b) a partial nominalistic interpretation $\mathrm{I}_N = (D_N, N_i)$ of L, where D_N is the domain of concreta and N_i are the nominalistic relations (which interpret the nominalistic predicates).

Let us call L_N the language without variables ranging over abstacta. Then:

> Def 1: An L-structure M is *nominalitically correct* iff its reduct to L_N is isomorphic to I_N.
> Def 2: A theory T is *weakly nominalistically adequate* iff all of its L_N-theorems are true in I_N.
> Def 3: A theory T is *nominalistically adequate* iff T has a model M whose L_N-reduct is isomorphic to I_N.

One can then show:

> Theorem 1. If T is n-adequate, then T is weakly n-adequate.
> Theorem 2. There are theories T that are *weakly* n-adequate, despite being n-*inadequate*.

Ketland has presented these ideas in a paper titled *Nominalistic Adequacy*, delivered to the Aristotelian Society, 10 January 2011.

ily envisage a situation in which two (or more) theories T1 and T2 have exactly the same nominalistically-stated consequences but differ in their mathematical formulations. These theories are n-equivalent. To simplify matters let us assume that theories have two distinct and separate (or separable) parts, one nominalistic (call it N) and another mathematical (call it M). So a theory T is in effect N+M.

Suppose we take NSR to accept, as it surely should, the view that it is not a necessary truth that mathematical objects do *not* exist. Take, then, a theory T1 (= N+M) and another theory T2 (= N +(−M)). T2, in effect, asserts that there are no mathematical objects at all and equates the content of the theory with its n-content. T1 and T2 are n-equivalent. Yet, given that there could be mathematical objects, there is a possible world W1 in which there are mathematical entities and in this possible world T1 is true and not just n-adequate, while T2 is n-adequate but false. Similarly, there is a possible world W2, in which there are no mathematical objects, in which T2 is n-adequate *and* true. How can we tell whether the actual world @ is like W2 and not like W1? That is, how can we tell whether T2 is n-adequate and false as opposed to n-adequate and true? Given that we read the mathematical parts of our theories literally, as NSR agrees, @ could be like either W1 or W2 and, if anything, it is a contingent matter what it is like. The advocate of NSR simply lacks the resources to make all these distinctions and, in particular, to discriminate between all these worlds. It follows that NSR cannot simply assert that @ is like W2; nor can it assert that theories are n-adequate and false as opposed to n-adequate and true (in the sense that there are no mathematical entities). Unless there is an argument to the effect that necessarily, mathematical entities do not exist, the advocate of NSR can at best be an *agnostic* about their existence. Note that an appeal to Ockham's razor in this context would be question-begging. Given that n-adequacy underdetermines truth and that n-adequacy is all we have, the issue at stake is precisely to offer reasons to apply Ockham's razor to mathematical entities as opposed to remaining agnostic about the reality.

Here is another problem. Take T1 (= N+M) and T2 (= N+M′) such that they are n-equivalent. Since, according to NSR, there are no mathematical entities, T1 and T2 are both false. But there are two ways in which a theory can be false—one is when there are no mathematical entities and the other is when it asserts something false about a putative mathematical entity. So to say that 3 is composite is false on both counts, but to say that 3 is prime is false only on the first count. Envisage a situation in which T1 and T2 are such that a claim of the sort '3 is composite' is part of T2 and a claim of the sort '3 is prime' is part of T1.[14]

There is something deeply wrong with T2 but an advocate of NSR should tolerate it because it has no bearing on the nominalistic adequacy of T2 and its presumed n-equivalence with T1. A standard riposte by nominalists (when a similar story is told about pure mathematics) is that '3 is prime' is true-in-the-story-of-mathematics, while '3 is composite' is false-in-the-story-of-mathematics. This kind of answer, whatever its merits in the case of pure mathematics, has *no* relevance to the present situation. If what matters is nominalistic adequacy and both theories

[14]This example is, of course, merely illustrative of the general point. Other more serious examples can be easily found, e.g., related to non-Euclidean geometries.

are n-adequate, it is irrelevant that one of them is true-in-the-story-of-mathematics while the other is not, since, on the NSR view, truth-in-the-story-of-mathematics has *no* bearing on truth-in-the-story-of-physics, that is on n-adequacy. What follows from this problem is that there is a sense in which NSR cannot respect even the role of mathematics in science that NSR finds unobjectionable. Mathematics, of the standard variety, is not even theoretically and descriptively indispensable, since NSR cannot discriminate between false mathematical theories and those that are standardly used by mathematicians and physicists.

Here is yet another problem. Take T1 (= N+M) and T2 (= N+M′) such that they are n-equivalent. For NSR, that's all than can be said of them. Any choice between them has no further epistemic relevance. But suppose that T1 (= N+M) is simpler, or more unified than T2(= N+M′). Suppose, that is, that the mathematical formulation M of T1 endows T1 with a number of theoretical virtues over the mathematical formulation M' of T2. For a scientific realist, theoretical virtues are truth-conducive. Hence, a scientific realist would have reasons to prefer T1 over T2 and to claim that T1 is more likely to be true than T2. But since T1 (= N+M) and 2 (= N+M′) are n-equivalent, the respects in which T1 is more likely to be true than T2 should have to do with the mathematical content of T1, e.g., its abstract structural claims. Note that the kind of situation just envisaged cannot be circumvented by taking M and M' to be merely descriptive and representational devices. Any serious advocate of NSR would have to reformulate T1(= N+M) and 2 (= N+M′) in such a way that they are mathematics-free, and textitthen show that the reformulated T1 is simpler and more unified than the reformulated T2. There is no general reason to expect this to be the case. It will depend on the further axioms that are chosen and employed.[15]

A fully-fledged scientific theory is a theory proper. The claim that a theory T is n-adequate does not amount to presenting another theory. But let us accept, for the sake of the argument, that T_{na} is a theory: the nominalist-reduct of T. Take two theories T1 and T2 and conjoin them. T1&T2 will have extra non-nominalistic consequences and, in all probability, extra nominalistic ones. Put together, instead, T_{na1} and T_{na2}. For one, T_{na1}&T_{na2} might even be inconsistent (simply because they might have contradictory platonistic parts; generally, T1 is n-adequate & T2 is n-adequate does not imply that T1&T2 is n-adequate.) But let us leave this to one side. Is there a guarantee that T_{na1}&T_{na2} has exactly all and only nominalistic is consequences of T1&T2? The only way to secure this is via the conservativeness of mathematics. If mathematics is indeed conservative, then all and only n-consequences that follow from (T1&T2) + M will follow from T_{na1}&T_{na2}. There is nothing wrong with conservativeness *per se*. But the pertinent point is that any possible benefits from going for nominalistic adequacy of scientific theories instead of their truth requires the conservativeness of mathematics; the move to n-adequacy

[15] Juha Saatsi (2007: 27) misses the point when he claims that "no mathematical entity has ever been *introduced* as the best explanation of some (mathematical or physical) phenomena". This claim seems oblivious to the facts that not all explanation in science is causal; that some explanations are unifications; and that some unifications—the most interesting ones—are effected only by introducing mathematical *structures*. It's irrelevant that these structures might not have themselves first been introduced "as the best explanation of some phenomena" (2007: 27). How an entity is first introduced is independent of the explanatory work it does, after it has been introduced.

doesn't add much to whatever benefits already follow from conservativeness.[16] Leng (2005: 76-7) has stressed that commitment to n-adequacy provided an easier route to nominalism than Field's reliance on nominalisation-plus-the-conservativeness-of-mathematics. If I am right, Leng's claim is wrong.

One further noteable reason to question the very idea of n-adequacy is that theories do not confront the phenomena (that is, *physical* occurrences) directly. Rather, they confront *models* of the data, which are mathematical structures (or more generally, NMAOs). The path of a planet, being an *ellipse*, is already a model of the data (and, in particular, a mathematical entity). The actual physical path is too messy to be of any use in the physical theory. Newton's theory accounts for the model of the data, that is the elliptic orbit (at least in the first instance). More generally, what really happens when a theory is applied to the world is that the theory is *first* applied to a model of the data, or to a heavily idealised (and mathematised) abstract physical object and *then* it is claimed that this model of the data captures some physical structure. As French (1999) has forcefully argued, inter-theoretic relations as well as relations between the theory and the world are ultimately, relations among mathematical structures.

Actually, this is not *very* surprising. Even if not all representation in science is based on isomorphism (or some other kind of morphism) a lot is so based and the very idea of structural similarity requires comparison between structures. But then, what is really compared when the n-adequacy of a theory is judged are two models, viz., two mathematical structures: the theoretical model and the model of the data. It is a further and separate claim that the model of the data (or the theoretical model for that matter) adequately represents concrete causal physical systems (or patterns). For the theory to be n-adequate it is the latter claim that has to be true. But this simply pushes the problem one step back. For now, the question is whether the model of the data itself (let's fix our attention on this to make things easier) is n-adequate vis-a-vis the appearances or the phenomena and answering this question presupposes either a direct confrontation of the model with the (unstructured) phenomena or the comparison of the model with another—that which (presumably) captures the *causal structure* of the phenomena. The first option does not seem to make much sense. The second option requires that the phenomena (or the world) has a built-in causal structure.

The key point here is not that this last assumption can be questioned.[17] Rather it is that the friends of NSR should come up with a conception of *the* causal *structure* of the world which is nominalist-friendly. This is too big an issue to be broached

[16] In his (1996) James Hawthorne has proved that if scientific theories are properly fleshed out, then they will not contain excess non-mathematical content, and in particular that no excess non-mathematical (that is, nominalistic) content will be generated when they are conjoined with other such theories. But his prove holds under very special conditions, which require that there are representation theorems between a mathematical theory T1 and a non-mathematical theory T2 such that a) every sentence of T2 is a non-mathematical consequence of T1 and b) adding set theory to T2 yields all and only the consequences of the mathematical version of T1. As Hawthorne notes, this kind of proof cannot be general and does not follow from the conservativeness of set theory. That these representation theorems hold has to be proved individually for each and every pair of theories. It is obvious that this kind of strategy cannot be helpful to NSR.

[17] For more on this, see Van Fraassen (2006) and my (2006).

here. In the next section, I will simply try to motivate the thought that there is more in the world than just causal structure.

6 Mixed facts revisited

It was stressed quite early on, in Section 2, that marrying scientific realism with mathematical realism requires commitment to mixed facts—that is facts that are constituted by a combination of concrete and abstract objects. To fix our ideas, let us concentrate on Balaguer's (1998: 133) example. Take the mixed statement:

(A) The physical system S is at forty degrees Celsius.

Taken at face value, (A) expresses a *mixed* fact, viz., that the physical system S stands in the Celsius relation to the number 40. But the number 40 is causally inert. Hence, Balaguer argues, if (A) is true (as we presume it is), it is made true by two sets of facts that are independent of each other: a physical fact concerning the temperature of S and a platonistic fact involving the number 40. (A), Balaguer notes, does not express a *bottom-level* mixed fact; the truth that (A) expresses supervenes on more basic facts that are *not* mixed: a purely physical fact and a purely mathematical fact. But this suggests that (A) has a nominalistic content "that captures its complete picture of S: that content is just that S holds its end of the '(A) bargain', that is, S does its part in making (A) true" (1998: 133). If this is the situation, Balaguer concludes, it can be the case that there are physical facts of the sort needed to make an empirical statement true, but no mathematical facts.

But is this the situation? Note, for a start, that what kind of facts make (A) true is not entirely clear. There is some physical fact, but in all probability, it has to do with the kinetic energy of S—a thing that, as Balaguer (1998: 133) adamantly admits, is not expressed by (A). So: the truth-maker of (A) is *some* physical state or other. Whatever that state is (if (A) is literally understood and true), it is such that it stands to a certain relation (the Celsius relation) to number 40. This number is, so to speak, the only non-negotiable part of the truth-maker of (A). What the physical state that stands in the Celsius relation to 40 is might vary (at least, we can be ignorant of it) but *that* it stands to this relation to *this* number is fixed (if (A) is true). It seems that the unity of the truth-maker of (A) requires this number, while it simply requires that there is *some or other* physical state related to this number, *modulo* the Celsius relation.[18] This unity is *not* a causal unity. It is not claimed that the mixed truth-maker is such that there is a *causal* relation C(S, 40) between the temperature-state of S (whatever that is) and 40. When it is asserted that the temperature-of-S-in-Celsius is 40 [(T_S-in-Celsius)=40] it is claimed that what is true of S concerning its temperature-in-Celsius is that it is equal to 40 and that this is true independently of what exactly it is that physically realises the temperature-state of S.

If we take '40' to be simply the name of a certain temperature state (we could have used the term 'Ralph' instead, as Balaguer notes in a different place), then all that (A) should be taken to express is that system S is in a certain temperature state

[18]This is a very important qualification, since I take to heart the Fregean lesson that the way numbers are attached to concrete objects is mediated by concepts (better: modes of presentation of these objects).

which is designated by '40'. But then we implicitly admit that (A) has a purely nominalistic content and that it has no platonistic content. In other words, we do not read (A) literally; we do not honestly assume that it expresses a mixed fact (even if it is not a basic mixed fact). That S captures the complete picture that (A) paints of the physical world is simply causation-begging—it equates completeness with causal completeness.

Actually, the point just made generalises. In a lot of mixed statements, e.g., of the form 'Physical system S is (or forms, or constitutes) a group' (B), the abstract part of the truth-maker (e.g., the group) captures (again in a non-causal way) general structural features of the specific physical system as well as what a number of such systems (despite their diversity in physical terms) share in common. As Weyl put it, group theory reveals "the essential features which are not contingent on a special form of the dynamical laws nor on special assumptions concerning the forces involved" (quoted by French 1999: 194).

Having said all this, I do not think that the facts that render statements like (A) are basic or bottom-level (though they are mixed). Clearly, that C(S, 40) is not bottom-level because it is a particular fact; and if we did admit it as bottom-level, we would have to make the massively uneconomical move to admit an infinity of bottom-level facts. Statements like (B) above are closer to being bottom-level, since they unify disparate physical systems. But in my view, bottom-level mixed facts are general facts that have to do with the structure of quantitative domains, the symmetry principles that characterise the world and things like that.

Balaguer is puzzled over the existence of mixed (physico-mathematical) facts. As has been noted, he takes it that mixed facts supervene on two independent sets of facts: physical *and* mathematical. Still, it does not follow that mathematical facts are dispensable *qua* parts of the truth-makers of mixed statements. Take the very simple statement 'The triangular road sign is yellow' (C). For this mixed statement to be true, a mixed fact is required, a part of which (so to speak) had to do with shapes and another with colours. There is no causal or other connection between shape-facts and colour-facts. We can even say that (C) has an S-content (whatever it asserts about shapes) and a C-content (whatever is asserts about colours). But though the two contents are independent of each other both are needed for the truth of (C), since even if each of them holds their end of the (C) bargain, none of them suffices to make (C) true.

7 A Diagnosis and a Conjecture

I have argued that NSR faces a number of problems in its attempt to motivate the weaker-than-full-truth notion of nominalistic adequacy. Even if we were to grant a clear and tolerably explained notion of n-adequacy, it would not follow that it would offer a better explanation of the success of science than the full truth of scientific theories. As noted already, discarding the abstract content of scientific theories (including the mathematical one) from being part of the best explanation of the success of theories is question-begging: it requires identifying explanation with causal explanation. The abstract content of theories plays a key role in ensuring the generality of the explanations offered and the unification of disparate phenomena in theoretical models. All this means that there is need for a more nuanced account of NMA (and of inference to the best explanation), where causal considerations are just one set out of many explanatory considerations. In my past writings (see my 1999, chapter 4) I too have put an emphasis on causal explanation. This has been wrong, especially insofar as it was meant to be exclusive of non-causal explanations. But clearly, not all explanation is causal—e.g., the explanation of low-level laws by reference to high-level ones. And explanation can also be of more abstract features of a system. Hence, even if causal explanation is indispensable, there is a more general level where the whole of the theory, with its abstract panoply, is seen as offering the best explanation.

There is a lot of hostility to abstract objects. Given their causal inertness, it is understandable—but, as I claimed, unjustified. But given this hostility, the friends of NSR take the view that the relevance of mathematics to empirical science has to do with our *understanding* and *representation* of the physical world and not with the operation of the physical world. If this is so, it can even be conceded that mathematical objects are theoretically and epistemologically indispensable, though, of course, metaphysically dispensable.

My own view is that Anti-Nominalistic Scientific Realism has to go all the way in its attack on NSR and claim that mathematical objects are part of the fabric of reality—though not in a way that has a causal impact on it. This implies that there are bottom-level mixed physico-mathematical facts. Actually, their existence seems to best explain the theoretical and epistemic indispensability of mathematics. But my conjecture is that though—to speak metaphorically—the concrete and the abstract co-operate to render mixed statements true, we are ignorant as to how this is done: as to what kind of unity a mixed physico-mathematical truth-maker has and in virtue of what it is united. More strongly, my conjecture is that we are cognitively closed to this kind of aspect of reality. I do not know how to back this up, but it seems to me it is the natural outcome of two predicaments: the first is that we tend to think of causation as the cement of the universe; the second is that, on reflection, we realise that the model of causal glue is too limited to account for the unity there is in the world—including the internal unity of the facts that make it up.[19]

[19]Versions of this paper have been presented in: the *PSA08* conference in Pittsburgh (November 2008); the *Metaphysics of Science* conference in the University of Melbourne (June 2009); and seminars in the Universities of Bristol (January 2009), Münster (April 2009), Barcelona (April 2010) and Milan–Bicocca (May 2010). Many thanks to audiences at these places for questions and comments—and especially to: Alexander Bird, Richard Boyd, Jim Brown, Mark Colyvan, Jose Diez, Brian Ellis, Geoff Hellman, Carl Hoefer, James Ladyman, Federico Laudisa, Mary Leng, Øystein Linnebo, Jose Martinez, Howard Sankey, Oliver Scholz, Christian Suhm and Nino Zanghi. Chris Pincock and Jeff Ketland deserve special thanks for their generous intellectual help and encouragement. A shorter version of this paper has appeared in *Philosophy of Science* (December 2010).

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