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III
Carnap and Incommensurability*

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1. Introduction

Relatively recent work on Carnap, based on his published papers and books as well as on his unpublished correspondence and other material, has suggested that Carnap and Kuhn might not have been miles apart when it comes to the issue of theory-change (cf. Earman 1993; Irzik & Grunberg 1995). Two prevailing thoughts are that a) Kuhnian 'paradigms' might be taken to be very similar to Carnapian 'linguistic frameworks' (cf. Irzik & Grunberg 1995, 286) and b) Kuhnian 'incommensurability' between competing paradigms is consonant with Carnap's thesis that when a linguistic framework is replaced by another, there is a change of language and the analytic-synthetic distinction (which is supposed to separate the meaning-fixing from the fact-stating component of a language) needs to be redrawn within the new framework (cf. Irzik & Grunberg 1995, 300-1). Irzik and Grunberg have gone on to note that Carnap endorsed "semantic incommensurability" (op.cit., 286). They base their claim on the theses that a) Carnap endorsed meaning holism; b) Carnap endorsed the thesis of 'theory-ladenness of observation'. They are certainly right in saying that "without semantic holism semantic incommensurability would be groundless; without theory-ladenness it would be severely restricted to the theoretical terms" (op.cit., 293). But, I think, they are wrong in claiming that Carnap endorsed either meaning holism or the theory-ladenness of observation.

The aim of this paper is to show how Carnap avoided the alleged problem of incommensurability. Better put, Carnap's view about the language of science (the linguistic framework in which theories are cast) is such that this problem does not arise. Drawing on his published and unpublished material, I highlight some connections between his work on semantics (and in particular his method of intension and extension in his *Meaning and Necessity*) and his mature work on the structure of scientific theories, which was based on his re-invention of the Ramsey-sentences. I claim that a key thought of Carnap's is captured by what may be called 'the extensional identity thesis', which (briefly put) asserts that to each physical concept expressed in Carnap's language of science there corresponds an extensionally identical mathematical function. In fact, as his work on semantic makes clear, Carnap took it that the variables of the Ramsey-sentence take two kinds of values, value intensions and value extensions. This move paves the way for a cumulative element in theory-change. Towards the end of the paper, I sketch the relevance of Carnap's strategy to our current philosophical thinking about theory-change in science.

2. Against Meaning Holism (I)

Carnap's (1956) is a monumental attempt to safeguard semantic atomism against Quine's and Hempel's urgings that semantic holism is inevitable. Though in the mid 1950s Carnap was still struggling with the explication of analyticity for the language of science, he thought he could make progress in two other fronts that he took them to be crucial for the development of an empiricist criterion of cognitive significance: a) the defence of a theory of the atomistic significance of theoretical terms, and b) the drawing of a boundary between the meaningful and the meaningless, i.e., finding a way to characterise isolated sentences (cf. 1956, 39-40). In Wilfrid Sellars's apt wording, Carnap attempted to forge a criterion for empirical meaningfulness that "would submit all descriptive terms in a theory to empirical control, and yet permit the theory to have surplus value" (cf. Sellars: "The Meaningfulness of Theoretical Terms"; 29 September 1955; Carnap Archive, Doc. 089-34-10, 2).

Later on, we shall have the opportunity to describe in detail Carnap's language L of science. For the time being, let us remind the reader that Carnap took this language L to consist of two parts: L_O, which is the observational sub-language whose variables range over observable things and their observable properties and relations, and L_T, which is the theoretical sub-language. On this two-tier model a V_T-term is meaningful not just in case it is part of a theory. It's meaningful iff when a sentence that contains it as the sole V_T-term is added to the theoretical postulates, this sentence makes some positive contribution to the experiential output of the theory. This move, Carnap thought, would render a term such as 'ψ -function' meaningful, although utterly remote from experience, but wouldn't thereby make meaningful *any* metaphysical speculation that is just tacked on to a scientific theory (cf. 1956, 39). Accordingly, a well-formed formula *S* of L_T is *meaningful* iff every descriptive constant of *S* is meaningful; and a sentence *S'* (or a postulate) of a theory T is *isolated*, if its omission from T does not affect the class of L_O-sentences that are deducible from T with the aid of correspondence rules (1956, 52-9). All in all, Carnap preferred to treat as meaningless terms that did not contribute to the empirical content of the theory, instead of adopting the (holistic) view that they acquired some meaning by 'fusion' with other meaningful terms (cf. 1956, 57).

The details of Carnap's criterion do not really matter here. Two general points, however, are worth noting. *First*, the criterion itself fails. This was pointed out by Hempel (1963). On the one hand, it is not necessary that all meaningful theoretical terms have straightforward experiential import, in the sense of contributing to the derivation of fresh observational consequences. Some theoretical terms may be introduced only in order to establish connections between other theoretical terms (e.g., the concept of 'asymptotic freedom' in quantum chromodynamics). It would be unwise to render such terms meaningless on the grounds that, either alone or in conjunction with other terms, they do not yield *extra* observational consequences. On the other hand, the criterion requires a deductive ordering, with respect to the postulates T and the correspondence rules C, of all sentences that contain theoretical terms non-vacuously so that at least one sentence *S* with a single theoretical term *Q* yields observational consequences with the aid of *no* other sentence (other than T- and C-postulates, that is), while all the other sentences get their significance relative to *S* (cf. 1956, 51-2). But it is an illusion to think that there is a privileged theoretical term *Q* which is tied directly to experience, and hence gets its meaning from experience in a more direct way than all the others. The *second*, and more important, point is that on Carnap's criterion meaningfulness is to be judged relative to a theory TC and not in isolation from it (cf. 1956, 48). Hence, a term is meaningful, if at all, only relative to the T- and C-postulates of the theory. Doesn't this commit Carnap to meaning holism?

Carnap thought he was not committed to meaning holism because the question of significance is raised for each and every theoretical term of a theory individually and not for the vocabulary of the theory as a whole, even though the question is raised *relative* to a theory (1956, 51). This is certainly right. Carnap did not propose a criterion for the empirical meaningfulness of the whole theory. Rather, he suggested a criterion for the meaningfulness of each non-primitive theoretical term, given that each such term features in a well-defined theory, i.e., a partially interpreted calculus TC.

Still, one might think, this amounts to admission of defeat. How do the basic V_T -terms—the ones that feature in the T-postulates—get their own meaning, if not holistically? Instead of admitting that T-postulates bestow a holistic meaning on their constituent terms (as Sellars, for instance, suggested), Carnap toyed with the idea that the T-postulates "contain meaningless terms" (1956, 57). But how can the T-postulates be presupposed for the characterisation of empirical significance and yet themselves be meaningless? Carnap is uncharacteristically elusive here. One of his suggestions is that since the T-postulates are "stated as holding with physical necessity" they convey some empirical meaning to whatever follows from them, and therefore to themselves (op.cit., 58). Can this idea of physical necessity be expressed in, or captured by, the extensional language of science he has constructed? In the extensional Language II of his *The Logical Syntax of Language*, which is virtually the same as the language Carnap constructs in his (1956), Carnap takes it that logical necessity is captured by analyticity. Physical necessity might well be captured by the P-postulates of the language, viz., those physical postulates on the basis of which a theory of the world is constructed. T-postulates cannot be analytic, since they have synthetic consequences. They are like the P-postulates of the *Logical Syntax of Language*. But how can physical necessity be characterised other than as entailment by the T-postulates, which express the fundamental laws of nature? The suggestion that T-postulates are meaningful because they hold with physical necessity is, therefore, unsatisfactory. The very idea of explicating physical necessity *presupposes* that T-postulates are meaningful statements.

So: Can Carnap avoid semantic holism at least for the T-postulates? His considered judgement is that T-postulates have to be regarded significant insofar as i) they are well-formed formulas, and ii) all non-primitive V_T-terms satisfy his criterion of empirical significance (1956, 62). But that's not entirely satisfactory either. The T-postulates get their meaning in virtue of the fact that, together with the C-postulates, they contribute to the experiential output of all other theoretical statements of the theory. Taken in isolation of this network of T- and C-postulates, the T-postulates are simply uninterpreted well-formed formulas. They become significant when they are seen as part of such a network. (One could say here that the T-postulates define their terms *implicitly*.) Be that as it may, it would be unfair (and wrong) to say that Carnap was a meaning holist, or that he was committed to it. Although relative to a set of postulates TC, the issue of meaningfulness is still raised for each and every non-primitive V_T-term individually. Local (or postulate) meaning holism seems to be a much more adequate characterisation of his position.

Irzik and Grunberg (1995, 293) argue that Carnap endorsed the claim that the meaning of observational terms was theory-dependent. Here, there is a simple and straightforward objection. Carnap (1952; 1974, 261-4) took the concept of analyticity to be entirely unproblematic for an observational language. He therefore thought the meanings of observational terms are fixed by analytic semantic rules. So, it is not theory that informs their meaning, nor indeed any synthetic truths about the world. It is this very fact that made him insist that the comparison of theories at the observational level is possible.

Irzik and Grunberg are not justified in concluding that Carnap endorsed (or, worse, independently accepted) Kuhn's thesis that competing paradigms in physics are incommensurable.[1] To be fair to them (and to Carnap) he did admit that there will be meaning changes "when a radical revolution in the system of science is made, especially by the introduction of a new primitive term and the addition of postulates for such term" (1956, 51). This claim, however, does not entail radical meaning variance—worse, incommensurability. It does not follow (as it does on Kuhn's holistic theory of meaning) that *any*, even the slightest, change in the theoretical web will result in meaning-change. In a sense, Carnap's ability to resist radical meaning-change is the outcome of his view that V_T -terms are incompletely interpreted and

open-ended. What Carnap means by that is that the meaning of a V_T -term can always be further specified by the addition of new T- and C-postulates. Insofar as no contradictory additions are made, this process does not change the meaning of T-terms; it only refines it.

Should we then stop here? What I want to argue in the remaining sections is that, irrespective of issues of meaning holism, the problem of incommensurability does not arise for Carnap. This is due to a central element in his late thought, viz., that there should be a language (or a 'linguistic framework') such that a) different physical theories can be expressed in it and b) new physical concepts can always find a place in it. His understanding of the language of science had all the necessary resources not just to answer charges of incommensurability, but to leave no space for a *coherent* formulation of the problem. The remaining sections will try to articulate this element of cumulativism in Carnap's thought. In the final section, I will sketch its possible significance for our current philosophical thinking about theory-change in science.

3. Ramsey-Sentences and the Language of Science

As is well-known, in the late 1950s Carnap made extensive use of the Ramsey-sentence in an attempt to characterise scientific theories.[2] What was less well-known until recently (see, for instance, my 1999, 2000a, 2000b, 2006) is that Carnap re-invented the Ramsey-sentence approach in an attempt to capture in a structural way the content of scientific theories.[3] In particular, Carnap thought that the Ramsey-sentence approach could steer a neutral course in the realism-instrumentalism debate. Given that the Ramsey-sentence of a theory does not easily admit of an instrumentalist reading (especially if the latter involves a *denial* of any existential implications on the part of a scientific theory), if Carnap wanted to hold on to his neutralism, it was quite pressing for him to dissociate the Ramsey-sentence from a straightforward realist reading. The relevant details have been presented in detail elsewhere (cf. Psillos 1999, chapter 3; 2000a). But the gist of his move was to read the Ramsey-sentence in a way that avoided existential commitments to unobservable entities. In a rather impressive move, Carnap took the variables of the Ramsey-sentence to range over *mathematical entities*. Where the Ramsey-sentence says there are non-empty classes of entities that are related to observable entities by the logico-mathematical relations given in the original theory, Carnap suggests that one is at liberty to think of these classes as classes of "mathematical objects".

To see how Carnap arrived at this seemingly strange position, we need to take in his way of constructing the language of science. As noted in the previous section, this language L consists of two parts: L_O, the observational sub-language whose variables range over observable things and their observable properties and relations, and L_T, the theoretical sub-language. The structure of L_T is very rich (cf. Carnap 1956; 1958). It is based on a type-theoretic logic with an infinite sequence of domains

D^0, D^1, D^2,..., where D^{n+1} is the power set of D^n. Each variable and each constant of L_T is assigned to a definite level. D^0 can be thought of as the domain of natural numbers—and so on. Thus constructed, L_T contains the expressive resources of classical mathematics. Carnap's invocation of such a strong structure for the language of science had to do with the fact that it has a certain theoretical advantage: all physical concepts that occur in theories can be shown to be represented by elements of D^1.[4] L_T can accommodate a space-time co-ordinate system such that each space-time point is assigned a 4-tuple of numbers. Physical magnitudes are introduced as functions from space-time points (quadruples of numbers) to numerical values. Physical objects are represented as four-dimensional regions inside which certain physical magnitudes have a certain distribution. Since each physical entity is shown to correspond to a suitable mathematical function in L_T, Carnap thinks it is but a short step to take *these* mathematical functions to be part of the content—in particular, the extension—of the corresponding physical (descriptive) designators. Although theoretical terms (V_T -terms) and predicates do occur in L_T, since L_T *is* a language for physical theories, the extensions of these terms need not, for Carnap, be taken to be entities of a new sort; rather they are the familiar entities of mathematics. As he (1958, 81) noted: "it is not necessary to assume new sorts of objects for the descriptive terms of theoretical physics". V_T -terms and predicates can be thought of as designating "mathematical objects" which, however, are physically characterised "so that they have the relations to the observable processes established by the C-postulates [i.e. the "correspondence postulates"] while simultaneously satisfying the conditions given in the T-postulates [i.e., the "theoretical postulates"]" (1958, 81). By way of example, Carnap noted that although descriptive, the constant 'n_p', defined as 'the cardinal number of planets', designates a natural number, which belongs to the domain D^0. The number n_p is identical with the number 9, yet the identity statement 'n_p=9' is synthetic: the world contributes to deciding whether it is true.

There are a quite few interpretative issues that I will skip over. The point I wish to stress is that Carnap should not be taken simply to assert that descriptive constants can refer to mathematical objects—a point already emphasised by Gottlob Frege. His point is much more exciting: in L_T "to each physical concept, let's say a function [Carnap means 'physical magnitude], there is an extensionally identical mathematical function" (Carnap Archive; Philosophical Foundations of Physics; Lecture XIV, 42, 111-23-01). Consequently, the structure of L_T makes it possible that mathematical entities are the extensions of corresponding descriptive designators. Note also that the language in which the Ramsey-sentences are stated is, for Carnap, none other than L (cf. 1974, 253). All observational content of the Ramsey-sentences is expressed in sub-language L_O and all non-observational content of the Ramsey-sentence is expressed by means of the sub-language L_T. So, it should be no surprise to see Carnap noting that "the Ramsey-sentence does indeed refer to theoretical entities by the use of abstract variables", but to immediately add: "[T]hese entities are not

unobservable physical objects like atoms, electrons, etc., but rather [at least in the form of the theoretical language which I have chosen in [1956] § VII] purely logico-mathematical entities, e.g., natural numbers, classes of such, classes of classes, etc." (1963, 963). Read in this fashion, the Ramsey-sentence says that

> the observable events in the world are such that there are numbers, classes of such etc., which are correlated with the events in a prescribed way and which have among themselves certain relations; and this assertion is clearly a factual statement about the world (ibid.).[5]

Let me call this claim 'the extensional identity thesis' (EIT). This is the position Carnap has developed. Briefly put, it claims that to each physical concept there corresponds in L_T an extensionally identical mathematical function. It may be thought that EIT is too strong, since it may be taken to imply the implausible view that the content of a physical designator is exhausted by a suitable mathematical entity—its extension.[6] But, this objection would be unfair. Partly in reply to a similar complaint made by Feigl in correspondence, Carnap supplemented the 'extensional identity' thesis with a substantive thesis of 'intensional difference'. In his reply to Feigl (Carnap to Feigl, August 4 1958—102-07-05), he explains:

> [T]he entities to which the variables in the Ramsey-sentence refer, are characterised not purely logically, but in a descriptive way; and this is the essential point. These entities are identical with mathematical entities only in the customary extensional way of speaking; see my example in square brackets on p.10. [Carnap refers to the example n_p: 'the cardinal number of planets'.] In an intensional language (in my own thinking I use mostly one of this kind) there is an important difference between the intension 9 and the intension n_p. The former is L-determinate [...], the latter is not. Thus, if by 'logical' or 'mathematical' we mean 'L-determinate', then the entities to which the variables in the Ramsey-sentence refer, are not logical.

Carnap's own semantic theory was characteristically two-dimensional. According to *Meaning and Necessity* (1947), expressions (be they constants, or predicates) are ascribed an intension *and* an extension—so for instance, the expression 'Human' has both an intension (the property of being human) and an extension (the class of humans). This method of intension and extension Carnap favours over the more traditional one of *naming*. The notion of L-determinateness is explained in his (1947, chapter II) and aims to capture the difference between descriptive and logical designators. A designator is L-determinate in a language L iff the semantical rules of L alone, without additional factual knowledge, determine its extension.[7] It is only derivatively that we can speak of L-determinate intensions. An L-determinate intension is the intension common to all those designators that are logically equivalent to a certain L-determinate designator. An L-determinate intension "is such that it conveys to us its extension" (1947, 89). What Carnap says is that '9' and n_p are different in that '9' is an L-determinate designator, while n_p is not. The extension of '9' is given by the semantic rules of L and it is the class of all classes which are equinumerous to 9. But n_p: 'the cardinal number of planets' is L-indeterminate because the semantic rules of L do not give its extension—finding its extension requires factual information. So '9' and n_p differ in intension.[8] The statement 'n_p=9' is a true identity statement. But it is not a logically true statement. It is a synthetic statement, and hence contingently true. In other words, although it is true that 'n_p=9', n_p is not necessarily equal to 9. Hence, n_p and '9' are co-extensional but different: they have different intensions.[9]

Carnap's full position is that when it comes to theoretical designators (expressions), both their intension and their extension should be taken into account. Once we make room for intensions, the problem of the range of the variables of the Ramsey-sentence gets resolved.[10] In Carnap's own method of extension and intension, variables are allowed *two* interpretations, taking intensional as well as extensional values (e.g., properties as well as classes, or individual concepts as well as individuals). Though in his (1947), Carnap does not yet have any use for the Ramsey-sentences, he discusses the case of second-order existential sentences $\exists f(..f..)$, which result from an existential generalisation over predicates (cf. 1947, 45). This is very much like a Ramsey-sentence. When such an existential generalisation is translated into the meta-language, it can have any of the following three forms:

There is an *f* such that ... *f*...
There is a class *f* such that ...*f*...
There is a property *f* such that ...*f*...

Carnap is adamant that since predicates have both an intension and an extension, the variable *f* should be taken to be "a variable both for classes and for properties" (ibid.). The variables have both value intensions and value extensions. We can then see that Carnap's method allows that the very same variables quantify over theoretical entities (he would rather say concepts), viz., the intensions of V_T -terms, as well as mathematical entities, viz., the extensions of V_T -terms—according to EIT.

Two questions arise at this point. *First*, in what sense are mathematical entities the extension of physical concepts? And second, isn't this "double-aspect ontology" an idle move, as Quine (1985, 329) notes? Answering the second question first will help us cast some light on the first question.

4. Detour via Intensions

Quine is quite right in saying that "the variables could be characterised more simply and no less adequately as admitting just intensional values" (ibid.). But this

is precisely Carnap's intention: the so-called "double-aspect" ontology is just "two forms of speech which can ultimately be reduced to one", i.e., to talk about intensions (1947, 91). The very advantage of the concept of L-determinacy is that it enables Carnap to perform a reduction of extensions to intensions. The reduction could not go the other way, the reason being that while intension can determine extension, if only the extension of an expression is given, its intension cannot be determined uniquely (1947, 112). Hence, the values of the variables of L_T will be theoretical entities (concepts).[11] What answer can we then give to the first question above, viz., in what sense are the extensions of V_T -terms characterised mathematically?

The answer is found in Carnap's way of reducing extension to intension. Here we need to get clear on Carnap's idea of intensional equivalence and its difference from intensional identity. As noted already, Carnap (1947, 23ff) points out that, strictly speaking, we can only talk about equivalent *designators* and not about equivalent intensions: two designators (e.g., predicates) are co-extensional (they have the same extension) iff they are equivalent; they are co-intensional iff they are L-equivalent. 'Equivalence' is understood in the usual way. Two predicates *P* and *Q* are equivalent iff it is true that $\forall x\,(Px \leftrightarrow Qx)$. Two predicates are L-equivalent iff $\forall x\,(Px \leftrightarrow Qx)$ is L-true (in a given language). As Carnap notes, there is also a tendency to transfer the notion of equivalence to intensions and to extensions themselves and to talk about equivalent extensions and equivalent intensions. In an extensional language, extensional equivalence *is* extensional identity. In an intensional language, there is an interesting difference between equivalent intensions and identical ones. We can (somewhat loosely) talk about equivalent intensions (that is, properties), which are not identical intensions (properties). Take, for instance, the properties *Featherless Biped* and *Human*. They have identical extensions (contingently)—since it is true that 'For all x (Human(x) $\leftrightarrow$ Biped(x))'. They are also equivalent in *intension* since the foregoing statement is true (i.e., it is true that 'Featherless Biped iff Human'). But the properties *Rational Animal* and *Human* are not merely equivalent in intension; they are identical: they have the same intension. This is because the statement 'Rational Animal iff Human' is L-true (in a given language). (As noted above, it would be more proper to say that the two *designators* are L-equivalent.) In light of this, we can say the following. Take two expressions (designators) *P* and *M*. If 'P iff M' is true, the two expressions are equivalent—which means that they have identical extensions and equivalent intensions. (Compare: 'Human' and 'Featherless Biped'.) If 'P iff M' is L-true, the two expressions are L-equivalent—which means that they have identical extensions and identical intensions. (Compare: 'Human' and 'Rational Animal'.)

Here is how Carnap reduces extension to intension. As noted already, in Carnap's method of extension and intension, the notion of L-determinate intension is such that an L-determinate intension *conveys* its extension. An L-determinate intension is the intension of an L-determinate designator, where the latter is such that its extension is determined solely by the semantic rules of the language. Note, Carnap claims, that though the relation between extension and intension is one-to-many, among the many intensions with a given extension "there is exactly one L-determinate intension, which may, in a way, be regarded as the representative of this extension" (1947, 89). This is not crystal clear, but to get the gist of Carnap's view let us look at expressions such as 'the number of planets', '9', 'the number of months in a (normal) pregnancy' and others like them. They are such that they have identical extensions and equivalent intensions. Among them, however, only one is L-determinate, viz., '9'. In fact, the intension of '9' is the *only* L-determinate intension which is equivalent to the intension of 'n_p' (and to 'the number of months in a pregnancy'). Carnap then says that the extension of a designator is defined as "the one L-determinate intension which is equivalent to the intension of the designator" (1947, 91). (In our example, the extension of 'n_p' is defined as the intension of '9').[12]

The above process of reduction of extension to intension requires finding for each descriptive designator 'P' another designator 'M' with an equivalent (but not identical) L-determinate intension. It requires, that is, finding true synthetic statements of the form 'P iff M', where 'M' is L-determinate. This is not something to be taken for granted. Carnap examined only simple artificial languages (like a co-ordinate language, in which the individuals are positions in an ordered domain (e.g., the domain of natural numbers)—see 1947, 74-5). Even then, he was clear that the method of L-determinacy does not apply unless the language has a certain structure (*like* a co-ordinate language) and the domain of discourse a certain order (*like* a progression with an initial point but no ending point). So, the requirement that to every extension there corresponds *exactly one* L-determinate intension is far from trivial.[13]

Why then should we accept that each descriptive designator (e.g., a predicate) is such that there is a unique L-determinate designator corresponding to it? To answer this, we need to link Carnap's method of intension and extension with his work on the structure of the language of science. It is extremely interesting that the language for which Carnap's controversial claim was suggested has the basic structure which he later on attributed to the language of scientific theories, that is L_T. This language, which in his (1947) he called the "co-ordinate language" Sp and employed it to skectch how the extensions of predicates and individual expressions can be reduced to their intensions, is precisely (part of) the language of physics L_T, as this is developed in his (1956; 1958).[14] There should be no surprise here, in hindsight. L_T, as we have seen, is such that it can supply an extensionally identical mathematical designator to each descriptive physical designator (recall the extensional identity thesis—EIT). Assuming that all these mathematical designators are L-determinate, (an assumption that Carnap never questioned), statements of the form 'P iff M', where 'P' stands for a physical designator and 'M' stands for an L-determinate mathematical designator, give, in the meta-language, the extensions of the corresponding physical designators. Better put, statements of the form 'P iff M', which are true but not L-true, show how the extension of a physical designator can be *represented* by the extension of a

corresponding mathematical designator. Since 'M' is an L-determinate designator, it conveys its extension. Hence, by virtue of the equivalence 'P iff M', the extension of 'P' is also conveyed. In a certain sense, however, we have now reached the converse of Carnap's original point: instead of having variables ranging over mathematical entities physically characterised, when we switch to an intensional language the variables range over physical entities (the intensions of physical designators) whose extensions are represented by extensionally identical mathematical designators.

Although Carnap never publicly brought together his views on Ramsey-sentences with his work on intension and extension, his appeal precisely to this link in his reply to Feigl's letter above suggests that the foregoing interpretation is legitimate. In any case, it is the kind of move that explains why he never bothered with the issue of incommensurability between theories. To put the point in a nutshell, this "dual aspect" interpretation of the variables of the Ramsey-sentence allowed him to envisage a linguistic framework in which all physical concepts can be expressed and compared with, even translated into, other physical concepts.

5. Why not Incommensurability?

It's tempting to see Carnap's claim that the extensions of descriptive terms are mathematical entities as a mere (and weird) artefact of his system without any independent motivation. There is a deep reason why Carnap insisted on a characterisation of the language of science in which the 'extensional identity' thesis holds: if all this is taken in, the problem of incommensurability does not arise. Before we see this, let us make a short digression to get a (loose) grip on the Kuhnian notion of incommensurability.

The Kuhnian notion of incommensurability stands for many things and has developed during Kuhn's own philosophical development. But two seem to be its central components (perhaps, in historical order): a) what Kuhn called "the fundamental aspect of incommensurability", that is that "the proponents of competing paradigms practice their trades in different worlds"; and b) the idea of some fundamental untranslatability between theories. Kuhn has offered different glosses of the second component. But he seems to have focused his thought on the view that "the claim that two theories are incommensurable is then the claim that there is no language, neutral or otherwise, into which both theories, conceived as a set of sentences, can be translated without residue or loss" (1983, 670). Later on, he supplemented this notion of untranslatability with his notion of "lexical structure". Two theories are incommensurable if their lexical structures (that is, their taxonomies of natural kinds) cannot be mapped into each other. When competing paradigm have locally different lexical structures, their incommensurability is local rather than global.

Conditions such as the above do not (and perhaps cannot) arise in a Carnapian linguistic framework such as L_T. If this sounds too strong, the least that can be said is that given Carnap's framework, incommensurability is not a necessary feature of the transition from an old to a new system of physical concepts and cannot hamper theory-comparison and choice. In fact, one of the independent attractions of Carnap's system was precisely that the extensional identity thesis made possible a) the translation of all physical concepts into L_T; and b) the translation of apparently distinct concepts into each other, by virtue of their extensionally identical mathematical designators. On this reading of Carnap, the practitioners of competing theories do *not* practice their trade in different words, since their theories are, ultimately, expressible in a common language L (that is L_O and L_T).

It is important to stress that it's too quick–and unwarranted–to conflate Carnap's use of 'linguistic framework' with his use of 'scientific theory'. If the two were the same, any wholesale change of theory would end up with the creation of a new language and the problem of how the two languages (the old and the new) could be translated into each other would acquire some force. It would be at least in principle possible to argue, along Kuhnian lines, that each language creates its own world (or that it structures the world in its own way). But for Carnap, a theory is a construction within the language of science, that is L_T together with an already interpreted "observational language" L_O (cf. 1956; 1958). A theory is expressed *within* L (i.e., L_T & L_O), and is a set T of theoretical axioms (the so-called theoretical- or T-postulates) and a set C of correspondence rules (or C-postulates) connecting the theoretical vocabulary V_T with the observational vocabulary V_O. So, two theories TC and TC' may differ in their theoretical terms, but their different terms will find their extensions in elements of L_T. For the structure of L_T is common to both of them. More generally, theory-change, that is change from TC to TC', even when it is wholesale, does not result in untranslatability (at least not necessarily). The extensional identity thesis makes possible that the concepts of the two theories may have the same extension, even if the intensions of the relevant terms/predicates have changed. The very fact that there is just one language of science in which all these changes occur, and the fact that this language has the resources to capture all new concepts by means of extensionally equivalent mathematical functions, makes translation possible (or to put it negatively, it does not make lack of translatability inevitable).

There has been a lot of work on whether incommensurability implies incomparability. What is certain, I think, is that translatability implies comparability. Hence, incomparability implies untranslatability. What Carnap's move in effect secures is that there cannot be conditions of general incomparability among theories. Hence, there cannot be conditions sufficient for untranslatability. Besides, Kuhn's dictum that "if two theories are incommensurable, they must be stated in mutually untranslatable languages" (1983, 669-670) would be the major premise of a Carnapian *modus tollens* of its antecedent, the minor premise being that the theories of physics are not stated in mutually untranslatable languages, but in one and the same language L_T.

To support all this with some textual evidence from Carnap, let me note the following. In devising L_T and in stressing the extensional identity thesis, one of

Carnap's major aims was to show that his framework for the analysis of the language of theories could be flexible enough to include new theoretical concepts that the future physicist might think up. In a characteristic passage of his (1958, 80), he noted:

> How should we construct a general conceptual scheme in which not only the object of an already given scheme of physics may fit, but also others, perhaps forces, particles, or special objects of an entirely new kind of which we presently have no conception but which a physicist might introduce tomorrow?

Carnap's insistence on the extensional identity between descriptive designators and mathematical ones aimed to address precisely this problem. When new physical concepts are introduced, the proposed linguistic framework can easily accommodate them because it can always provide the relevant extensionally identical mathematical designators. No matter what the features of a new physical magnitude may be, its logical type will be identical with a certain mathematical function, which can be expressed in L_T. In his letter to Feigl, Carnap says explicitly:

> My emphasis on the kind of variables had only the purpose to indicate that the logical types of the required variables are not of any strange new kind, but just of the kind we are familiar with in mathematics, say in a simple type hierarchy, beginning not with objects, but with natural numbers, as in my language II in Logical Syntax (Carnap Archive, 102-07-05).

When new entities (or concepts) are introduced, there is no need to change radically the linguistic framework in which scientific theories are developed. Even when theories employ different theoretical concepts, they can still be compared from an extensional point of view, by finding the mathematical functions that correspond to these concepts and by examining whether these are extensionally identical, i.e., whether they have the same values for all points on which they are defined. In other words, Carnap's main motivation was the construction of a stable logico-linguistic environment for the development of scientific theories. In his *Lecture Course on the Foundations of Physics*, Carnap makes this point explicitly:

> Thereby, I believe, we have entirely got rid of the problem how we can foresee the strange entities which physicists might introduce in the future. If you think of the theoretical entities as things of some kind which nobody has ever seen, like electrons or so, then you will think that we cannot foresee what strange kinds of things physicists will conjure up—we might not even be able to imagine them today. But if we assume that every physical theoretical term that will be introduced belongs to a certain type, then that type can be provided for. I think, even the system outlined above, containing all finite types,

will presumably be sufficient for all concepts of physics for quite some time" (Carnap Archive, 111-23-01) (cf. also 1966, 253).

To put the point in an anachronistic way, Carnap aimed to show that when new concepts are developed the "lexical structure" of the language of physics need not change, at least not in a fundamental (mathematical) level.

6. Analyticity and Incommensurability

It may be thought that the fact that the analytic-synthetic distinction needs to be redrawn when there is radical theory-change, a fact that Carnap always insisted on, makes translatability impossible. The thought here may be that the very fact that different sets of sentences count as analytic in different theories cannot allow for a translation between the different theories. This thought would be wrong. Let us see why.

One of the major attractions of the Ramsey-sentence approach was that it enabled Carnap to solve an elusive problem: how to define analyticity for a theoretical language (cf. 1958). The crux of the problem was that in the standard formulation of a scientific theory as the conjunction of a set T of theoretical postulates and a set C of correspondence rules, the meaning-fixing function and the fact-stating function of the theory were fused.[15] Carnap's re-invention of the Ramsey-sentence allowed him to solve this problem by noting that the theory in the old form (i.e., TC) is *logically equivalent* to the conjunction (^{R}TC & ($^{R}TC \rightarrow TC$)). ^{R}TC is the Ramsey-sentence of the theory, while the conditional $^{R}TC \rightarrow TC$—known as the Carnap sentence—asserts that *if* there is a class of entities that satisfy the Ramsey-sentence, *then* the t-terms of the theory denote the members of this class. Carnap suggested that the Ramsey-sentence of the theory captured its factual content (which was expressed in the rich language L_T), while the conditional $^{R}TC \rightarrow TC$ captured its analytic content (it is a *meaning postulate*). This is, Carnap noted, because the conditional $^{R}TC \rightarrow TC$ has no factual content: its own Ramsey-sentence, which would express its factual content if it had any, is logically true. He thereby thought that he solved the problem of "how to define A-truth [analytic truth] in the sense of analyticity or truth based on meaning for a theoretical language" (cf. Carnap 2000, 162).[16]

Carnap's view can be criticised on many grounds (cf. my 2006). But it *cannot* be criticised on the grounds that it leads to incommensurability. Recall that it is misleading to think of Carnap as adopting the view that the meaning of V_T-terms is characterised in a holistic way by the T- and C-postulates. Apart from everything else that was stated in section 2 in support of this claim, it should be added that Carnap insisted that the analytic (or meaning-fixing) part of the theory should be separated from the synthetic (or fact-stating) part. But he also admitted that this distinction could not be made if the theory were viewed as a conjunction of T- and C-postulates (cf. 1958, 82). The new logically equivalent formulation ^{R}TC & ($^{R}TC \rightarrow TC$) solves

both problems. The analytic part of the theory—the conditional $^{R}TC \rightarrow TC$—fixes the meaning of the V_T -terms by associating them with entities that realise the Ramsey-sentence of the theory. The synthetic part—the Ramsey-sentence ^{R}TC—captures not just the empirical content of the theory but also an abstract claim of realisation (*there are entities* ...). Suppose that a new theory TC' is adopted. Associated with it will be a new meaning postulate of the form $^{R}TC' \rightarrow TC'$, and a new Ramsey-sentence $^{R}TC'$. The new meaning postulate may attach new terms to the entities that realise the Ramsey-sentence of the new theory. (Or, it may be argued that even if some terms remain the same, they in fact have different meanings, since they are introduced by means of a new meaning postulate.) Still, it is perfectly possible that the common terms of the two theories have the same extensions, that is that they are represented by the same mathematical functions. Or, that at least some of the terms of the old theory have the same extension as terms of the new theory. Be that as it may, the point here is that the very fact that, on Carnap's view, the extensions of the theoretical terms of the two theories will be expressible in a common language L_T undercuts the claim that the two theories are incommensurable. To say the least, even if they are not translatable into each other—and there is no reason why they should be; after all the new theory is supposed to make progress on the old—they are both translatable into L_T, and therefore 'commensurable' in their claims about nature. Here again the essential point is that possible differences in intension do not lead to incommensurability since there can still be an extensional (mathematical) comparison of the terms of the two theories.

7. Against Meaning Holism (II)

If there is a typical thesis associated with meaning holism surely it should be that the meaning of a term is determined in a holistic manner if this term cannot be explicitly defined by virtue of a few other terms of the language in which it occurs. As we have already seen, Carnap was far from accepting meaning holism. What needs to be added here is that after his re-invention of the Ramsey-sentence, and after putting it to work in an explication of analyticity for a theoretical language, he coupled all this with David Hilbert's ε-operator in an attempt to show how the analytic postulates of the theoretical language can offer *explicit definitions* of theoretical terms (cf. 1961; 2000b). The basic idea is the following.

Hilbert's ε-operator is defined by one axiom: $\exists y Fy \rightarrow F(\varepsilon_x Fx)$. This means that *if* anything has the property F, *then* the entity $\varepsilon_x Fx$ has this property. $\varepsilon_x Fx$ may be thought of as the *ε-representative* of the elements of a non-empty class F, without further specifying which element it is. Let the theoretical terms of the theory TC form an n-tuple $t=<t_1,...,t_n>$. Hilbert's ε-operator allows us to select an *arbitrary* class among the classes of entities that satisfy the theory such that the n-tuple *t* of theoretical terms designate this class. In other words, the n-tuple *t* of theoretical terms designates the ε-representative of the classes of entities that satisfy the theory. Then, each and every theoretical term of the n-tuple is explicitly defined as the ε-representative of the i-th member of the n-tuple. The theory TC is expressed in the following form:

$$^{R}TC \,\&\, A_O \,\&\, A'_T$$

where A_O is the conjunction of the analytic postulates of TC that are expressed in the observational language, and A'_T is the conjunction of n+1 explicit definitions of the *n* theoretical terms of TC.[17] The theory can still be split up into two parts, one analytic (A_O & A'_T), the other synthetic, viz., the Ramsey-sentence ^{R}TC of the theory. But the initial meaning postulate $^{R}TC \rightarrow TC$ is now replaced by n+1 explicit definitions of each and every one of the *n* theoretical terms of the theory.[18] Carnap (2000b) went on to show that this new way to characterise the analytic component of the theory logically implies the meaning postulate $^{R}TC \rightarrow TC$. So, the old characterisation of theory as the conjunction ^{R}TC & ($^{R}TC \rightarrow TC$) can be recovered with in the new ε-framework. The sole (but big) advantage of the new characterisation of the theory lies in the fact that it provides an explicit definition of each and every theoretical term of the theory.

Why did Carnap prefer the logically stronger version of the theory—the one based on the ε-operator? Because, I claim, he took it to make possible the restoration of full semantic atomism for theoretical terms. To be sure, each and every theoretical term is explicitly defined relative to the n-tuple *t* of the theoretical terms of the theory. Still, relative to this n-tuple, the meaning of each and every theoretical term of the theory can be fully disentangled from the meanings of the rest and be given by a single meaning postulate.

There is more to say. Carnap's use of the ε-operator contains the kernel of a theory of reference of the theoretical terms. Notice that Carnap's move resembles a strategy very common to mathematical reasoning, where something must be proved for an arbitrary entity of a kind. Every so often, a mathematician will say (something like the following): take a point between *A* and *B*; call it *C*; then ..., where the dots are replaced by a claim about C. When something like that is done, the point designated by 'C' is the ε-representative of a class of points for which a certain claim needs to be proved. Not only is it assumed that there is such a point, but also concrete things can be proved of it. The proof holds good not just for it, but also for any other point which the arbitrarily chosen—but named—point represents. Carnap's ε-operator transfers this pattern of reasoning to the case of theoretical entities. Where the Ramsey-sentence of the theory asserts that there are classes of entities that satisfy the theory, the ε-operator method takes the terms of the theory to refer to *some* such class: the ε-representative of the class. Yet, it is not merely the existence of this class is asserted. This class is also designated: it is the class to which the theoretical terms of the theory refer and this class should be seen as the ε-representative of the classes of entities

that satisfy the theory. So, this class—the ε-representative of the extensions of the theoretical terms—is the reference of the theoretical terms of the theory. This way of talking about the reference of theoretical terms has a certain advantage for Carnap. Carnap's method achieves two things: a) it makes possible the further specification of the exact reference of theoretical terms; and b) it shows how there can be (in principle, at least) some referential continuity in theory change.

If we think, as we should, of a theory in a process of growth, its basic entities (that is the entities to which the theoretical terms refer) should be able to enjoy further specification of their role, their connection with other entities. All this can be done by adding further postulates and correspondence rules to the theory. Hilbert's device makes it possible that, when all these are added, the reference of the theoretical terms does not change. It's still the ε-representative of the classes of entities which realise the theory, but now this ε-representative is further specified. As Carnap noted, what is special about Hilbert's operator is that it is an "indeterminate constant". Unlike the ι-operator, it does not assert the existence of a unique class that realises the theory. If it did, when new postulates were added to the theory, the reference of the t-terms would have to change: the unique class which was taken to realise the theory TC would give way to another unique class which would now be taken to realise the new theory TC'. With the ε-operator, it is perfectly consistent to argue that the class that was the ε-representative of the entities that realise TC is the same as the class of entities that realise TC', the only difference being that in TC' this class is further specified. So, Carnap noted, his ε-definition gives "just so much specification as we can give, and not more. We do not want to give more because the meaning should be left unspecified in some respect, because otherwise the physicist could not—as he wants to—add tomorrow more and more postulates, and even more and more correspondence postulates, and thereby make the meaning of the same term more specific than {it is} today". And he concluded: "it seems to me that the ε-operator is just exactly the tailored-made tool that we needed, in order to give an explicit definition, that, in spite of being explicit, does not determine the meaning completely, but just to the extent that is needed" (2000b, 171-2).

In the ε-calculus, the so-called uniqueness (or ι-)operator (the equivalent of the definite article) can be easily defined: if there exists only one entity satisfying Fx, then '$\varepsilon_x Fx$' is to be read as 'the entity having the property F'. So, the ε-operator characterises an indefinite description, whereas the ι-operator characterises a definite one. This is exactly were Carnap's approach differs from Lewis's. In his (1970), David Lewis modified Carnap's approach based on the ι-operator. He therefore insisted on the uniqueness requirement. He suggested that if there was no unique realisation of the theory, the theoretical terms should be considered denotationless. His motivation for this claim was that such a view is the lesser of two evils. In case of non-unique realisation, there is no non-arbitrary way to pick one realisation. So, Lewis thought, we are forced either to accept that theoretical terms do not name anything, or that they name the elements of one arbitrarily chosen realisation. For him, however, "either of these alternatives concedes too much to the instrumentalist view of a theory as a mere formal abacus" (1970, 432).[19]

Perhaps Carnap wanted to leave the door to instrumentalism open. But for methodological purposes, his way of understanding reference is more fruitful. Apart from being able to view a theory in a process of growth, Carnap's use of the ε-operator allows him to show how there can be referential continuity in theory-change. In line with his extensional identity thesis, he may take the theoretical terms of the theory to refer to either some physical entities or to their extensionally identical mathematical functions. Both of these classes can be taken to be the extensions of theoretical terms of the theory, but the ε-operator does not pick out one of them. All it does is to speak of their extension in a more abstract way, by means of an ε-representative. What is thereby gained is quite significant. Taking some mathematical functions to be the ε-representatives of the extensions of the theoretical terms, Carnap could show how terms of a new and an old theory can have the same reference: by comparing the mathematically specified ε-representatives of the extensions of their terms. Here again, there is no guarantee that there should be term-by-term translation. But, a) this is not in principle impossible; and b) when it happens, there is a clear sense in which the new theory can be seen as an attempt to further specify the reference of the old theory's theoretical terms.

8. Contemporary Lessons

So far, I have explained in detail how *Carnap* managed to avoid the problem of incommensurability. But I think this story does not have a merely historical significance. Carnap's insight may be suitably developed so that it offers a useful way to address the alleged incommensurability problem, in general. In this final section, I shall sketch how this can be done.

Ordinary philosophical talk treats the extension of an expression as being an individual, or an object, or a quantity, or more generally, something concrete. To be sure, extensions of predicates are taken to be sets—hence, abstract entities. But even then, we have in mind a set of concrete entities, e.g., a set of electrons, or, a physical quantity mass being the extension of the term 'mass'. This ordinary talk seems to be well-motivated. But it would seem to clash with Carnap's talk of mathematical entities as extensions of physical concepts. This clash might be taken to be enough to detract from Carnap's insight. But there needn't be any clash. Thinking of the extensions of physical concepts (or of theoretical terms), one can introduce a distinction between the physical entity to which the concept refers and an extensionally identical mathematical entity. Call the first *P-extension* and the second *M-extension*. To borrow (and modify) one of Carnap's unpublished examples, one can talk of the P-extension of the 'electric field vector' (or 'E') as a physical quantity responsible for certain electric and magnetic effects and of the M-extension of the very same concept as a mathematical function *f* of a certain logical type, that is, a function from quadruples of reals to triples of reals. Given a Carnap-like rich language L_T, we know that there is such a mathematical function *f* which is extensionally identical with *E*, i.e., *E* and *f* have the same value for any argument: for any x_1, x_2, x_3, t, $E(x_1, x_2, x_3, t)=f(x_1, x_2, x_3, t)$. The very possibility of such a function allows the quantification of the physical magnitude that is the P-extension of E. It allows, that is, the construction of a mathematical theory that describes electrical and magnetic phenomena, a theory that yields quantitative laws and predictions. In saying all this, one need not take literally Carnap's extensional identity thesis. We may take it (and its implications) quasi-literally: accept that physical concepts have concrete physical entities as their putative referents, but also allow that these putative referents can be *represented* by mathematical entities, their M-extensions.

If this is done, the resulting theory of reference is rich enough to account for reference-continuity and reference-comparison. Instead of positing a 'naked' causal agent as the referent of a theoretical term, as the pure causal theories would have it, this quasi-Carnapian theory of reference would ground the causal agent to which the term refers to a certain mathematical description. (Alternatively put, it would *represent* the P-extension of a term by means of its M-extension.) So, when it comes to issues of judging sameness of reference, there can be at least a substantive necessary condition: two terms which feature in two different theories of the same domain refer to the same entity only if their corresponding M-extensions are identical (or, more weakly, suitably connected by means of mathematical descriptions). Here again, the intensions of the terms might well have changed. But the claim of referential continuity is grounded in a richer framework; in particular, a framework that takes account of how theoretical terms are represented in the mathematical structure of the theories.

All this needs further development. But for the time being, I only want to note that the possibility of such a theory seems intelligible—and perhaps, hitherto unnoticed. Perhaps a damning objection might be the following: this view might be intelligible when it comes to physical magnitude terms, such as 'mass' or 'electric field vector'; but what about 'electron'? For, the objection implies, 'electron' is a general term which refers—putatively—to entities with no corresponding M-extension. So, the view sketched might just collapse. Considering this objection, there are two things to note. *First*, it might well be the case that the view I am exploring is restricted to physical magnitude terms, where it makes sense to identify a corresponding mathematical function. *Second*, however, it may be also able to cover general terms such as 'electron'. In any serious physical theory about, say, electrons, the electrons will be associated with a mathematical description of their causal-nomological role. One can then look at this description, or a central part thereof, and attempt to see whether subsequent theories of apparently the same entities (electrons) have incorporated this mathematical description. If there is some continuity at this level, at least the problem of comparability of the two conceptions of electrons is resolved. This proposal, if intelligible at all, should be taken to be essentially Carnapian.

Notes

* An ancestor of this paper was presented a few years ago in a conference titled 'Incommensurability and Related Matters' in Hanover, June 1999. I would like to thank the participants for their comments, but particularly: Richard Boyd, Paul Hoyningen-Huene and Howard Sankey. Bill Demopoulos deserves special thanks for very useful comments and for encouraging me not to let this paper stay in a drawer for ever. There is no better occasion for me to present this paper than this—in a special volume in honour of Nikos Avgelis, who brought Carnap, Schlick and co. to Greece and with them a breath of fresh air in contemporary Greek philosophy. It is to him that I dedicate this work. All Carnap archival material is quoted with the permission of Pittsburgh University. All rights reserved. All Feigl archival material is quoted with the permission of the Minnesota Center for the Philosophy of Science. All rights reserved.

[1] Carnap never, to the best of my knowledge, employed the term "paradigm", nor the term "incommensurability".

[2] To remind the reader: in order to get the Ramsey-sentence ${}^{R}TC$ of a (finitely axiomatisable) theory TC we conjoin the axioms of TC in a single sentence, replace all theoretical predicates with distinct variables u_i, and then bind these variables by placing an equal number of existential quantifiers $\exists u_i$ in front of the resulting formula. Suppose that the theory TC is represented as $TC\,(t_1,\dots,t_n;\, o_1,\dots,o_m)$, where TC is a purely logical m+n-predicate. The Ramsey-sentence ${}^{R}TC$ of TC is: $\exists u_1 \exists u_2 \dots \exists u_n TC\,(u_1,\dots,u_n;\, o_1,\dots,o_m)$. For simplicity let us say that the T-terms of TC form an n-tuple $t=<t_1,\dots,t_n>$, and the O-terms of TC form an m-tuple $o=<o_1,\dots,o_m>$. Then, ${}^{R}TC$ takes the more convenient form: $\exists u TC(u,o)$.

[3] It was Carl Hempel in *The Theoretician's Dilemma*, published in 1958, who coined the now famous expression 'Ramsey-sentence'. When Rudolf Carnap read a draft of Hempel's piece in 1956, he realised that he had *re-invented* Ramsey-sentences. Carnap had developed an "existentialised" form of scientific theory. In the protocol of a conference at Los Angeles, organised by Herbert Feigl in 1955, Carnap is reported to have extended the results obtained by William Craig to "type theory, (involving introducing theoretical terms as auxiliary constants standing for existentially generalised functional variables in 'long' sentence containing only observational terms as true constants)" (Feigl Archive, 04-172-02, 14). I have told this philosophical story in some detail in my (1999, chapter 3).

[4] Carnap (1956, 44) says they "can be shown belong to $D[{}^{1}]$".

[5] If more textual evidence is needed, here is another quote from Carnap's unpublished *Lectures on the Philosophical Foundations of Physics*: "we may understand the Ramsey sentence as speaking merely about mathematical entities in addition to the observational entities. This is the status of the Ramsey sentence from my point of view" (CA, Philosophical Foundations of Physics; Lecture XIV, 42, 111-23-01).

[6] Carnap himself noted this objection with humour. He said: "These ideas I expressed in an unpublished manuscript, which I wrote a few months ago. [Carnap refers here to what came to be his (1958).] Some of my friends who read it, said: 'For heaven's sake, what are you doing there? Are you turning all physics to mathematics? Is this now a strange schematised ontology of science? Does science speak only about mathematical entities? They speak about things, visible things and then electrons, electromagnetic fields, and other things tomorrow which we do not even think of today—you cannot call these entities just mathematical entities!" (Carnap Archive, Philosophical Foundations of Physics; Lecture XIV, 36, 111-23-01). Carnap went on to clarify his thesis and what follows in the text draws on his subsequent clarification.

[7] Carnap (1947, 72-3), to be sure, says that a designator is L-determinate in L iff the semantic rules of L suffice to *give* its extension, where this giving is to be contrasted with describing it. He then goes on to describe the structural properties a language must have so that its semantic rules *give* the extensions of designators.

[8] For Carnap, cardinal number expressions are predicates of second level; they have properties of second level (properties of properties) as intentions and classes of second level (classes of classes) as extensions.

[9] Carnap's approach is explained in detail in his lecture course on the Foundations of Physics 1958-59, (Lecture XIV, Carnap Archive, 111-23-01).

[10] Carnap adopts a fully objective, and in particular, anti-psychologistic account of intensions. The intensions of predicates, for instance, are properties where the latter are understood as "something physical that the things have, a side or aspect or component or character of things" (1947, 20; cf. also 22). Similarly for concepts: his term 'concept', which is used as "a common designation for properties, relations, and similar entities", refers to "something objective that is found in nature and that is expressed in language by a designator of nonsentential form" (1947, 21).

[11] In a passage where Carnap speaks of his method of extension and intension in general, he notes: "(...) a designator stands primarily for its intension; the intension is what is actually conveyed by the designator from the speaker to the listener, it is what the listener understands. The reference to the extension, on the other hand, is secondary; the extension concerns the location of application of the designator, so that, in general it cannot be determined by the listener merely on the basis of his understanding of the designator, but only with the help of factual knowledge. Therefore, if somebody insists on regarding a designator as a name either of its intension or of its extension, then the first would be more adequate (...) (1947, 157).

[12] To allay some possible fears here, it is obvious (almost) that Carnap can recover extensions (as they are standardly understood) out of the L-determinate intensions.

[13] As noted above, in his (1947), Carnap motivated this claim with a few examples. For instance, he noted that the one and only L-determinate intension which corresponds to the extension *True* is *L-True*.

[14] In his (1947, 79-80) there is only a brief sketch of how his method of L-determinacy applies to the language of physics. However, even then, he takes it that the language of physics must be seen as a structured language whose domain is space-time points and whose standard expressions are four standard real-number expressions—since each space-time point is determined by three space co-ordinates and one time co-ordinate.

[15] For a short but nice presentation of the problem, cf. Carnap (1974, 269).

[16] For a detailed account of all this, see my (2000b).

[17] Recall that the Ramsey-sentence ${}^{R}TC$ of the theory takes the form: $\exists u TC(u,o)$. Take a language L_ε that contains ε as a constant. Hilbert's axiom defines the designatum of an n-tuple *t* as the ε-representative of $TC(u,o)$. So, we can define: $t=\varepsilon_u TC(u,o)$. Since *t* is an n-tuple, we can then define each theoretical term t_i (i=1,...,n) as the i-th member of the n-tuple, using the schema: $t_i=\varepsilon_x[\exists u_1\, \exists u_2 \dots \exists u_n\, (t=<u_1,\dots,u_n>\ \&\ x=u_i)]$.

[18] There are *n* definitions for each of the *n* t-terms and one for the n-tuple.

[19] For a recent modification of Lewis's views which brings together the Ramsey-sentence approach with the thought that there is some vagueness associated with the meaning of theoretical concepts, see Papineau (1996).

References

Carnap, R. (1947) *Meaning and Necessity: A Study in Semantics and Modal Logic*, Third Enlarged Edition 1956, Chicago IL: The University of Chicago Press.

Carnap, R. (1952) 'Meaning Postulates', *Philosophical Studies* 3: 65-73—reprinted in *Meaning and Necessity: A Study in Semantics and Modal Logic*, (1956) Chicago IL: The University of Chicago Press.

Carnap, R. (1956) 'The Methodological Character of Theoretical Concepts' in H. Feigl & M. Scriven (eds) *The Foundations of Science and the Concepts of Psychology and Psychoanalysis*, Minnesota Studies in the Philosophy of Science, 1, Minneapolis: University of Minnesota Press, pp.38-76.

Carnap, R. (1958) 'Observation Language and Theoretical Language' in J. Hintikka (ed.) *Rudolf Carnap, Logical Empiricist* (1975), Dordrecht: Reidel—translation of 'Beobachtungssprache und Theoretische Sprache', *Dialectica* 12: 236-248.

Carnap, R. (1961) 'On the Use of Hilbert's ε-operator in Scientific Theories' in Bar-Hillel, Y. *et al.* (eds) *Essays on the Foundations of Mathematics*, Jerusalem: The Magnes Press, pp.156-164.

Carnap, R. (1963) 'Replies and Systematic Expositions' in Schilpp, P. (ed.) *The Philosophy of Rudolf Carnap*, La Salle IL: Open Court, pp.859-1013.

Carnap, R. (1974) *An Introduction to the Philosophy of Science*, New York: Basic Books

Carnap, R. (2000) 'Theoretical Concepts in Science' *Studies in History and Philosophy of Science* 31: 158-72.

Hempel, C. (1963) 'Implications of Carnap's work for the Philosophy of Science' in Schilpp, P. (ed.) *The Philosophy of Rudolf Carnap*, La Salle IL: Open Court, pp.685-709.

Irzik, G. & Grunberg, T. (1995) 'Carnap and Kuhn: Arch Enemies or Close Allies?' *British Journal for the Philosophy of Science* 46: 285-307.

Kuhn, T. S. (1983) 'Commensurability, Comparability, Communicability', in P. Asquith and T. Nickles (eds) *PSA 1982*, East Lansing, MI: The Philosophy of Science Association, pp.669-687.

Lewis, D. (1970) 'How to Define Theoretical Terms', *Journal of Philosophy* 67: 427-46.

Papineau, D. (1996) 'Theory-Dependent Concepts', *Philosophy of Science* 63:1-20.

Psillos, S. (1999) *Scientific Realism: How Science Tracks Truth*. London & New York: Routledge.

Psillos, S. (2000a) 'Carnap, the Ramsey-Sentence and Realistic Empiricism', *Erkenntnis* 52: 253-79.

Psillos, S. (2000b) 'An Introduction to Carnap's 'Theoretical Concepts in Science', *Studies in History and Philosophy of Science* 31: 151-58.

Psillos, S. (2006) "Ramsey's Ramsey-sentences", in M. C. Galavotti (ed.) *Cambridge and Vienna: Frank P Ramsey and the Vienna Circle (Vienna Circle Institute Yearbook 12)*, Springer. pp. 67-90

Quine, W. V. (1985) 'Carnap's Positivistic Travail', *Fundamenta Scientiae* 5: 325-33.