# On a Dynamical Mechanism Underlying the Intensification of Tropical Cyclones

**N.A. Bakas and P.J. Ioannou**

**Abstract** Tropical cyclones are among the most life threatening and destructive natural phenomena on Earth. A dynamical mechanism for cyclone intensification that has been proposed is based on the idea that patches of high vorticity associated with individual convective systems are quickly axisymmetrized, feeding their energy into the circular vortex. In this work, Stochastic Structural Stability Theory (SSST) is used to achieve a comprehensive understanding of this physical mechanism. According to SSST, the distribution of momentum fluxes arising from the field of asymmetric eddies associated with a given mean vortex structure, is obtained using a linear model of stochastic turbulence. The resulting momentum flux distribution is then coupled with the equation governing the evolution of the mean vortex to produce a closed set of eddy/mean vortex equations. We apply the SSST tools to a two dimensional, non-divergent model of stochastically forced asymmetric eddies. We show that the process intensifying a weak vortex is shearing of asymmetric eddies with small azimuthal scale that produces upgradient fluxes. For stochastic forcing with amplitude larger than a certain threshold, these upgradient fluxes lead to a structural instability of the eddy/mean vortex system and to an exponentially growing vortex.

## 1 Introduction

Tropical cyclones are among the most life threatening and destructive natural phenomena on Earth. However, despite our knowledge of the climatological conditions favoring tropical cyclogenesis, a widely accepted theory for tropical cyclone formation and intensification does not exist. Early theoretical work suggested that tropical depressions intensify by utilizing cooperative feedbacks

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between cumulus clouds and the large scale flow (Charney and Eliasen 1964), or between surface heat fluxes and the surface wind (Rotunno and Emanuel 1987). Another theory is based on the idea that patches of high vorticity associated with individual convective systems are quickly axisymmetrized, feeding their energy into the vortex scale flow (Montgomery and Enagonio 1998), a process that can be described in terms of the interaction between Vortex Rossby Waves (VRW) and the circular vortex (Montgomery and Kallenbach 1997).

The goal of this work is to provide a theory for the systematic organization of the turbulent fluxes intensifying the vortex, based on the interaction between VRW and the mean vortex and building on results from linear stochastic turbulence modeling that has led to a novel framework for investigating the organization of eddy fluxes in turbulent zonal flows called Stochastic Structural Stability Theory (SSST, Farrell and Ioannou 2003). In the context of SSST, the forcing of VRW, that can be traced to highly intermittent, short time scale processes (e.g. convection), is modeled stochastically (Farrell and Ioannou 1993a, b; DelSole and Farrell 1996). Furthermore, the VRW-VRW interactions are ignored, yielding a linear stochastic model for the VRW evolution. The resulting momentum flux distribution arising from the VRW is coupled with the mean vortex momentum equations to obtain a closed dynamical system for the joint evolution of the VRW statistics and the mean vortex. The dynamics of this system that will be derived in detail in Sect. 2, will be investigated in order to address whether axisymmetrization can lead to the transformation of a small depression into a tropical cyclone of sufficient intensity, focusing on the VRW-mean vortex dynamics leading to such intensification.

## 2 Evolution Equations for a Barotropic Vortex

Consider a forced non-divergent, flow in which the Coriolis parameter $f$ is set to a local, constant value. A streamfunction $\psi$ can be defined such that: $[u,v] = [-(1/r)\psi_\theta,\psi_r]$, where $u$, $v$ are the radial, $r$, and azimuthal, $\theta$, components of velocity. Since we are interested in the effect of asymmetric perturbations on a circular vortex, we decompose the streamfunction into an axi-symmetric component (indicated with upper case) and an asymmetric eddy component that is typically termed as Vortex Rossby Waves (VRW) (indicated with a tilde): $\psi(r,\theta,t) = \Psi(r,t) + \tilde{\psi}(r,\theta,t)$. Harmonic VRW of the form $\tilde{\psi}(r,\theta,t) = \psi'(r,t)$ $e^{im\theta}$, where $m$ is the azimuthal wavenumber, evolve according to:

$$\left(\partial_t + im\frac{V}{r}\right)\zeta' - \frac{1}{r}\frac{dZ}{dr} im\Delta^{-1}\zeta' = -\mu\zeta' + f_{ext} + f_{eddy} \tag{1}$$

where $V = \Psi_r$ is the azimuthal averaged tangential wind, $\zeta' = \Delta\psi'$ is the relative vorticity of the VRW, $Z = \Delta\Psi$ is the vorticity of the mean flow, $\Delta$ is the Laplacian in polar coordinates, $\mu$ is the coefficient of linear dissipation, $f_{ext}$, is the external forcing and $f_{eddy}$ is the forcing term from the eddy-eddy interactions. On the other hand, the circular vortex is forced by the radial vorticity fluxes and evolves as:

$$\partial_t V = -\overline{u'\zeta'} - \mu V \tag{2}$$

Following previous studies of stochastic turbulence modeling (Farrell and Ioannou 1993a, b; DelSole and Farrell 1996), we parameterize the eddy forcing term $f_{ext} + f_{eddy}$ as a stochastic process. We also discretize the differential operators with finite differences on a radial channel [$0,R$], imposing zero boundary conditions for the streamfunction at the boundaries. The operators then, become finite dimensional matrix approximations of the continuous operators and the variables $\zeta'$, $V$ become column vectors with elements the values of the variables at the grid points defined by the elements of the radial vector **r**. In matrix notation, (1) takes the form:

$$\frac{d\zeta}{dt} = \mathbf{A}\zeta + \mathbf{F}\xi \tag{3}$$

where the spatial structure of the forcing is given by the columns of **F** and $\boldsymbol{\xi}$ is a vector giving the time variation of the forcing. [*Notdefined*] is the matrix form of the linear dynamics of VRW:

$$\mathbf{A} = -imdiag(V)\mathbf{R}^{-1} - im\mathbf{R}^{-1}\mathbf{DZ}\Delta^{-1} - \mu\mathrm{I} \tag{4}$$

where $\mathbf{R} = \mathrm{diag}(\mathbf{r})$, **I** is the identity matrix and diag(●) denotes the diagonal matrix with diagonal elements the vector ●. The dynamics of VRW comprise of advection of the vorticity of VRW by the vortex, advection of the vorticity gradient **DZ** by VRW and dissipation. Similarly, (2) is written as:

$$\frac{d\mathbf{V}}{dt} = \mathbf{M} - \mu\mathbf{V} = -\frac{m}{2}\mathrm{Im}\left[\Delta^{-1}\zeta\zeta^{\dagger}\right] - \mu\mathbf{V} \tag{5}$$

where **M** are the radial vorticity fluxes, † denotes the Hermitian transpose and vecd denotes the operation of extracting the diagonal elements of a matrix. The random vector process $\xi$ has statistically independent elements and is a Gaussian white noise in time with zero mean and unit variance: $<\xi> = 0$, $<\xi_i \; \xi_j> = \delta_{ij}\delta(t-s)$, where the angle brackets denote an ensemble average over realizations of the forcing. The spatial localization of the excitation is dictated by the matrix **F** which is chosen to have $l$ columns and elements $F_{ij} = J_m(k_j r_i/R)$, where $J_m$ is the Bessel function of order $m$ and $k_j$ is the $j$th zero of $J_m$. This specification leads to a statistically homogeneous excitation with forcing that is coherent over a distance inversely proportional to $k_l$. Finally, the forcing is normalized to have input power $\varepsilon$.

The system of (3), (5) describes the dynamics of a single realization of the stochastically excited VRW interacting with the circular mean vortex. Assuming a large number of independent realizations of the forcing and taking an ensemble average of the excited wave fields, we obtain a deterministic equation governing the evolution of the ensemble average enstrophy covariance matrix $\mathbf{C} = <\boldsymbol{\zeta}\boldsymbol{\zeta}^{\dagger}>$:

$$\frac{d\mathbf{C}}{dt} = \mathbf{AC} + \mathbf{CA}^{\dagger} + \varepsilon\mathbf{Q} \tag{6}$$

where $\mathbf{Q} = \mathbf{FF}^{\dagger}$. Under an ergodic assumption, the ensemble average of the eddy vorticity fluxes is equal to the azimuthal mean vorticity fluxes and the vortex therefore evolves as:

$$\frac{d\mathbf{V}}{dt} = \mathbf{M} - \mu\mathbf{V} = -\frac{m}{2}\mathrm{Im}\left[\Delta^{-1}\mathbf{C}\right] - \mu\mathbf{V} \tag{7}$$

Equations (6), (7) form a deterministic, autonomous, nonlinear system for the evolution of the mean vortex under the influence of its consistent field of VRW. The fixed points $\mathbf{V}^E$ and $\mathbf{C}^E$, if they exist, define statistical equilibria in the presence of an eddy field with covariance $\mathbf{C}^E$. The stability of the VRW-vortex equilibria $\mathbf{V}^E$ and $\mathbf{C}^E$ can then be determined by considering the evolution of small perturbations $\delta\mathbf{V}$, $\delta\mathbf{C}$ about the equilibrium. Because of the operator Im in (6), we must write separate equations for the evolution of the real, $\delta\mathbf{C}^R$, and imaginary part, $\delta\mathbf{C}^I$, of the covariance. The resulting stability equations for the evolution of $\delta\mathbf{V}$, $\delta\mathbf{C}^R$, $\delta\mathbf{C}^I$ can be written in the form:

$$\frac{d}{dt}\begin{bmatrix} vec(\delta\mathbf{C}^R) \\ vec(\delta\mathbf{C}^I) \\ \delta\mathbf{V} \end{bmatrix} = \mathbf{L}\begin{bmatrix} vec(\delta\mathbf{C}^R) \\ vec(\delta\mathbf{C}^I) \\ \delta\mathbf{V} \end{bmatrix} \tag{8}$$

where vec is the vector representation of a matrix obtained by stacking sequentially the columns of a matrix on top of each other. The structural stability operator $\mathbf{L}$ determines the stability of the VRW-vortex equilibria.

## 3 Structural Instability for an Equilibrium with No Mean Vortex

The state with $\mathbf{V}^E = 0$ and $\mathbf{C}^E = \varepsilon\mathbf{Q}/2\,\mu$, is a fixed point of the system (5)–(6) and the goal is to determine the stability of this statistical equilibrium state. It can be readily shown that $\mathbf{L}$ has a large number of decaying eigenmodes with $\delta\mathbf{V} = 0$ that do not modify the mean vortex. The remaining eigenvalues are given by:

$$\lambda_n = -\frac{3\mu}{2} \pm \frac{1}{2}\sqrt{2\mu^2 + 4s_n^2}, n = 1, 2, \ldots, N \tag{9}$$

where $s_n$ are the eigenvalues of matrix $\mathbf{S}$ that determines the sensitivity of the vorticity fluxes to small changes in the vortex velocity $\mathbf{S} = \partial\mathbf{M}/\partial\mathbf{V}$. As a result, the

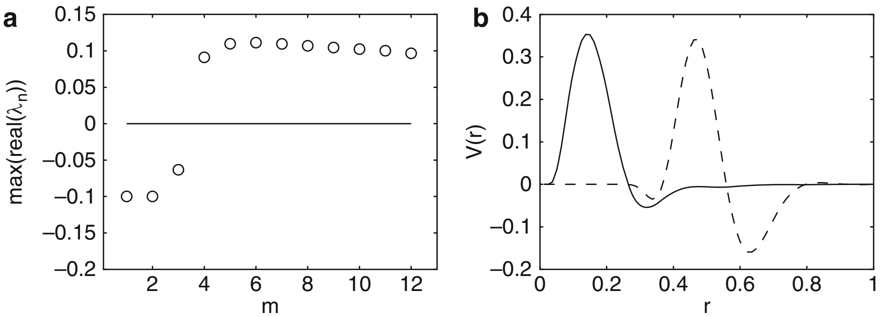


**Fig. 1** (**a**) Maximum growth rate as a function of azimuthal wavenumber $m$. (**b**) The tangential velocity of the most unstable eigenfunction for $m = 4$ (*solid line*) and $m = 12$ (*dashed line*). For both panels, the power input is $\varepsilon = 10^{-3}$, $\mu = 0.1$ and $l = 40$

state with no mean vortex becomes unstable and a vortex emerges and intensifies only if the eigenvalues of **S** are positive, that is only if the VRW are organized by the vortex perturbation in such a way to yield upgradient vorticity fluxes.

The sensitivity operator can be explicitly calculated in terms of **Q**, **R** and **Δ** and is given as the sum of two commuting operators $\mathbf{S} = \mathbf{S}^{ad} - \mathbf{S}^{vg}$. The first operator, $\mathbf{S}^{ad}$, determines the sensitivity of the fluxes to changes in the advection of the vorticity of VRW, and the second operator, $\mathbf{S}^{vg}$, determines the sensitivity of the fluxes to changes in the advection of the mean vorticity gradient by the VRW. The eigenvalues of $\mathbf{S}^{ad}$, $\mathbf{S}^{vg}$ were numerically calculated and were both found to be positive. Consequently, advection of the eddy vorticity by the perturbed vortex yields upgradient fluxes and is destabilizing, while advection of the perturbed mean vorticity gradient by the VRW yields downgradient fluxes and has a stabilizing tendency. For $m > 2$, the eigenvalues of $\mathbf{S}^{ad}$ are larger, resulting in an overall destabilizing tendency.

Calculation of the eigenvalues $\lambda_n$ revealed that another necessary condition for instability, is that the input power $\varepsilon$ should be above a certain threshold, so that the eddy forcing, as measured by $s_n$ can overcome the mean vortex dissipation. When the two necessary conditions are met, there is an emerging vortex whose mean tangential velocity grows exponentially. The maximum growth rate as a function of $m$ is shown in Fig. 1a for a given forcing and eddy dissipation and roughly saturates at a constant value for large $m$, showing that small scale VRW are the most unstable. Figure 1b shows the most unstable mean flow perturbation, for $m = 4$ and $m = 10$. We observe that as $m$ increases, the core of the vortex moves away from $r = 0$.

## 4 Conclusions

Axisymmetrization of VRW has been proposed as a dynamical mechanism for the intensification of a circular vortex. The VRW-circular vortex system is examined in this work within the framework of SSST. In the context of SSST, the average VRW field and the circular vortex form a coupled system, in which the evolution of VRW is obtained using a linear stochastic model and the resulting vorticity fluxes force the mean vortex. Using SSST, the structural stability of a vortex with no velocity, subjected to homogeneous stochastic excitation was examined. The eigenvalues of the linear operator governing the evolution of mean vortex perturbations and the associated VRW statistics around the equilibrium state were calculated. The structural stability was found to depend on the sensitivity of the vorticity fluxes to changes in the vortex. Calculation of the eigenvalues of the sensitivity operator revealed two mechanisms underlying the instability: shearing of VRW by the vortex that is destabilizing, and advection of the mean vorticity gradient by the VRW that is stabilizing. VRW with small (large) azimuthal scales were found to be destabilizing (stabilizing) and a threshold for the amplitude of the excitation was found, above which an infinitesimal vortex is intensified. The maximum growth rate saturates for small azimuthal scales. It occurs for a vortex, whose core is at a radial distance proportional to the azimuthal wavelength of the VRW.

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