



# NUMERICAL LINEAR ALGEBRA DAY

Thursday 10 November 2011

Department of Mathematics  
University of Athens

## Scientific Responsible

- Marilena Mitrouli

## Organization

- Grigorios Kounadis
- Paraskevi Fika

## Invited Speakers

- Claude Brezinski
- Efstratios Gallopoulos
- Apostolos Hadjidimos
- Nikolaos Missirlis
- Dimitrios Noutsos
- Paris Vassalos

## **ABSTRACTS**

“The PageRank problem”

**Claude Brezinski**

*Universite des Sciences et Technologies de Lille, France*

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First, we will explain what is the Pagerank problem for classifying the pages of the web according to their importance. Then, the algorithm for solving the problem will be presented, and its properties will be discussed. Finally, some improvements will be given.

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“Irreducibility and extensions of Ostrowski’s Theorem”

**Apostolos Hadjidimos**

*University of Thessaly, Volos*

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In this talk an extension of the Ostrowski’s Theorem for complex square irreducible matrices is presented. Also extensions of similar statements for square complex matrices are analyzed and completed. Most of the statements in this work cover also the case of reducible matrices.

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“Variations on PageRank’: Generalizing PageRank, multidamping and inhomogeneous matrix products”

**Efstratios Gallopoulos**

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Since its publication in 1998, PageRank has become a popular algorithm for link-based ranking both for “math and money”. PageRank is a special case of a larger class of “functional rankings”, that have been presented in the literature as matrix power series. We review an approach, we call multidamping, for re-interpreting methods for ranking web pages based on matrix power series representations functional rankings. We show that under certain conditions, matrix polynomials having as argument a normalized stochastic matrix can be expressed as inhomogeneous products of Google matrices. We analyze the spectral properties of these inhomogeneous products, investigate their convergence and present numerical experiments illustrating our findings.

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“Iterative methods for solving Laplacian systems for graphs”

**Nikolaos Missirlis**

*University of Athens*

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We consider iterative methods for solving Laplacian systems of graphs. In this context we study the Diffusion method for solving the Load Balancing problem.

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“Solution of ill conditioned Toeplitz systems by preconditioned conjugate gradient methods”

**Dimitrios Noutsos**

*University of Ioannina*

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Toeplitz systems arise in a variety of application in mathematics, scientific computing and engineering: image restoration problems in image processing; discretization of differential equations and convolution integral equations; time series analysis and control theory. Conjugate gradient methods are efficient for the solution of well conditioned Toeplitz systems. For ill conditioned Toeplitz systems, preconditioning is required. The usual efficient preconditioners are band Toeplitz matrices or matrices belonging to a class of matrix algebra, such as circulant, tau, or Hartley matrices or combinations of the classes above.

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“Trigonometric matrix algebras can be spectrally equivalent with ill-condition Toeplitz matrices”

**Paris Vassalos**

*Athens University of Economics and Business*

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For constructing fast and efficient iterative procedure or effective multigrid schemes, the study of the spectrum of the matrix sequences  $\{A_n\}_n = \{P_n^{-1}(f)T_n(f)\}$  is of great importance. In this talk, we will focus on the case where  $T_n(f)$  is a Toeplitz matrix generated by a nonnegative real function  $f$ , and  $P_n(f)$  denotes matrices belonging to tau or circulant algebras. Assuming that the symbol  $f$  has discrete roots of non integer order, we will show that under suitable assumptions the spectrum of the matrix sequence  $\{A_n\}_n$  is bounded by constants far away from zero and infinity.

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“Solving typical numerical analysis problems using parallel computing capabilities”

**MATLAB Workshop**

*Mentor Hellas*

[zg@mentorhellas.com](mailto:zg@mentorhellas.com)

This presentation includes two parts: In the first part you will see how can you use MATLAB to solve typical numerical analysis problems including basic linear algebra, ordinary differential equations and optimization problems. In the second part, we will demonstrate how to use MATLAB parallel tools to solve the above problems by taking advantage of the latest multiprocessing systems.

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“The Good Pivots property of Hadamard Matrices and related issues”

**Christos Kravvaritis**

*Technical University of Munich, Germany*

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The notion of good pivots is introduced for the first time and its importance for the study of the growth problem for Hadamard matrices is examined. Specifically, Hadamard matrices possessing this property satisfy Cryer's conjecture with equality, namely their growth factor is equal to their order. Thus, the conjecture can be settled for specific orders up to H-equivalence. Theoretical results about good pivots and pivot patterns of Hadamard matrices are given. The new tools are utilized for demonstrating that the growth factor of a Hadamard matrix of order 20 is 20, up to H-equivalence. All possible values of the sixth and seventh pivot, resp. leading principal minor of order 6 and 7, of any CP Hadamard matrix are specified explicitly for the first time.

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“Different approaches evaluating minors for Hadamard matrices”

**Anna Karapiperi**

*University of Athens*

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In the present talk we give three different proofs for the evaluation of the  $n-2$  minor of an Hadamard matrix. The first proof was given in 1907 by Sharpe and is based on the notion of the cofactor. The next proof is a constructive one proposed in 2006 by Kravvaritis and Mitrouli who focused on the special structure of an Hadamard matrix and the basic properties of such a matrix. At the end we propose a new approach using eigenvalues. The ideas that will be presented can be applied in any  $n-j$ ,  $j \geq 1$ , minor.

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“Preconditioning augmented linear systems”

**Maria Louka**

*University of Athens*

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In this paper we study various types of preconditioning for large sparse augmented linear systems. We form the corresponding iterative methods and study their convergence analysis. In particular, for each method we find sufficient conditions for convergence and determine the optimum values of its parameters.

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“Numerical Linear Algebra methods for image denoising and deblurring”

**Dimitrios Triantafyllou**

*Hellenic Army Academy*

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In this presentation two hybrid methods for denoising and deblurring a 2-D blurred image are presented. Our procedures use Sylvester matrices and numerical linear algebra techniques in order to restore the initial image in an efficient way. The methods can handle measurement errors and noise as well, using an approximate approach in blind deconvolution process. Examples illustrating procedures are given.

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“An extrapolation method estimating the trace of the inverse of matrices”

**Paraskevi Fika**

*University of Athens*

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Let  $A$  be a symmetric positive definite matrix. Using some mathematical tools, we will see how we can obtain estimates for the trace of the inverse of this matrix, by an extrapolation procedure on its moments. Finally, we will see how we can extend these estimates in the case of a general matrix  $A$ .

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“Some projection methods for estimating the diagonal of a matrix inverse”

**Vasilis Kalantzis**

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The estimation of the diagonal a matrix inverse is an important problem in many applications. Following the statistical estimation approach of Hutchinson and

more recent advances these methods depend on the presence of an effective method for computing  $z' A^{-1} z$  for a suite of appropriately designed  $z'$  s. Recently, Bekas et al., combined some of these techniques with conjugate gradient, iterative refinement and parallel processing to produce a method for that returned orders of magnitude better performance on very large matrices. In this talk we present results from the application of solvers based on projections that exploit the right-hand sides to further accelerate the overall computation.

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“The nine neighbor Extrapolated Diffusion method for weighted torus graphs”

**Aikaterini Dimitrakopoulou**

*University of Athens*

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The convergence analysis of the Extrapolated Diffusion (EDF) was developed in [1] and [2] for the weighted torus and mesh graphs, respectively using the set  $N_1(i)$  of nearest neighbors of a node  $i$  in the graph. In the present work we propose a Diffusion scheme which employs the set  $N_1(i) \cup N_2(i)$ , where  $N_2(i)$  denotes the four neighbors of node  $i$  with path length two (see Figure 1) in order to increase the convergence rate. We study the convergence analysis of the new Diffusion scheme with nine neighbors (NEDF) for weighted torus graphs. In particular, we find closed form formulae for the optimum values of the edge weights and the extrapolation parameter. A 60% increase in the convergence rate of NEDF compared to the conventional EDF method is shown analytically and numerically.

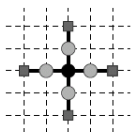


Fig. 1.  $\circ$  denotes neighbors with path length one,  $\square$  denotes cross-shape neighbors with path length two.

#### References

1. G. Karagiorgos and N. M. Missirlis. Convergence of the diffusion method for weighted torus graphusing fourier analysis. Th. Comp. Science, 401:1-16, 2008.
2. G. S. Markomanolis and N. M. Missirlis. Optimum diffusion for load balancing in mesh networks. Euro-Par 2010, 230-241, September 2010.

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“Approximation techniques for cloud motion simulation on CUDA”

**Stulianos Kondulis**

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Graphics Cards present a new powerful tool for High Performance & Scientific Computing. In this talk we use the Navier-Stokes equation solvers available in the CUDA parallel computing platform of NVIDIA in order to provide approximation techniques for cloud simulation. We do this by means of particle

simulation techniques. The original cloud image is created by using a Perlin noise function. Following initialization, the movement of particles is simulated by solving the Navier-Stokes equations and modifying the particles' coordinates accordingly. A blur function is then applied to create a homogenized cloud image. We illustrate our technique with numerical simulations.

*Melencolia I* (The British Museum, Burton 1989) is a 1514 engraving by the German Renaissance master Albrecht Dürer. It is an allegorical composition which has been the subject of many interpretations. The engraving shows a disorganized jumble of scientific equipment lying unused while an intellectual sits absorbed in thought. Dürer's magic square is located in the upper right-hand corner of the engraving.

The date of 1514 appears in two middle cells of the bottom row of the magic square, as well as above Dürer's monogram at bottom right. This 4x4 magic square, as well as having traditional magic square rules, its four quadrants, corners and centers equal the same number, 34, which happens to belong to the Fibonacci sequence.



# Program

- 9:15-9:30 Opening Remarks  
9:30-10:00 **Claude Brezinski**, *Université des Sciences et Technologies de Lille, France.*  
“The PageRank problem”  
10:00-10:30 **Apostolos Hadjidimos**, *University of Thessaly, Volos.*  
“Irreducibility and extensions of Ostrowski’s Theorem”  
10:30-11:00 **Efstratios Gallopoulos**, *University of Patras.*  
“Variations on PageRank’: Generalizing PageRank, multidamping and inhomogeneous matrix products”
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COFFEE BREAK

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- 11:30-12:00 **Nikolaos Missirlis**, *University of Athens.*  
“Iterative methods for solving Laplacian systems for graphs”  
12:00-12:30 **Dimitrios Noutsos**, *University of Ioannina.*  
“Solution of ill conditioned Toeplitz systems by preconditioned conjugate gradient methods”  
12:30-13:00 **Paris Vassalos**, *Athens University of Economics and Business.*  
“Trigonometric matrix algebras can be spectrally equivalent with ill-condition Toeplitz matrices”
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- 13:30-15:30 **MATLAB Workshop**  
Solving typical numerical analysis problems  
using parallel computing capabilities.

**MENTOR Hellas**  
Scientific Engineering Software

**Stulianos Kondulis, Janis Kalofolias**, *University of Patras.*  
“Towards a Spike implementation on the MATLAB Parallel Computing Toolbox”

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- 16:00-16:20 **Christos Kravvaritis**, *Technical University of Munich, Germany.*  
“The good pivots property of Hadamard Matrices and related issues”  
16:20-16:40 **Anna Karapiperi**, *University of Athens.*  
“Different approaches evaluating minors for Hadamard matrices”  
16:40-17:00 **Maria Louka**, *University of Athens.*  
“Preconditioning augmented linear systems”  
17:00-17:20 **Dimitrios Triantafyllou**, *Hellenic Army Academy.*  
“Numerical Linear Algebra methods for image denoising and deblurring”
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COFFEE BREAK

## Software Presentation

- 17:45-17:55 **Ioannis Vassilopoulos**, *University of Athens.*  
“Computing pivot patterns of Hadamard matrices”  
17:55-18:05 **Eugenia-Maria Kontopoulou**, *University of Patras.*  
“TMG: A MATLAB tool for text mining”
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- 18:15-18:35 **Paraskevi Fika**, *University of Athens.*  
“An extrapolation method estimating the trace of the inverse of matrices”  
18:35-18:55 **Vasilis Kalantzis**, *University of Patras.*  
“Some projection methods for estimating the diagonal of a matrix inverse”  
18:55-19:15 **Aikaterini Dimitrakopoulou**, *University of Athens.*  
“The nine neighbor Extrapolated Diffusion method for weighted torus graphs”  
19:15-19:35 **Stulianos Kondulis**, *University of Patras.*  
“Approximation techniques for cloud motion simulation on CUDA”