

①

$$3(x-2y) - 2(-4y - 5(2x-3y) + 4xy) - 3(y-2x) =$$

$$= 3x - 6y - 2(-4y - 10x + 15y + 4xy) - 3y + 6x =$$

$$= 3x - 6y - 2(11y - 10x + 4xy) - 3y + 6x =$$

$$= 3x - 6y - 22y + 20x - 8xy - 3y + 6x =$$

$$= 29x - 31y - 8xy = 29 \cdot (-1) - 31 \cdot \frac{2}{3} - 8 \cdot (-1) \cdot \frac{2}{3} =$$

$$= -29 - \frac{62}{3} + \frac{16}{3} = -29 - \frac{46}{3} = \frac{-87 - 46}{3} = -\frac{133}{3} \quad \checkmark$$

②

$$\text{i)} \frac{3}{2} - \frac{1}{5+\frac{1}{2}} = \frac{3}{2} - \frac{1}{\frac{11}{2}} = \frac{\frac{1}{2}}{\frac{3}{2}} - \frac{\frac{2}{2}}{11} = \frac{29}{22} \quad \checkmark$$

$$\text{ii)} \frac{\frac{1}{2} - \frac{2}{3}}{1 + \frac{1}{2 - \frac{1}{2}}} = \frac{\frac{1}{3}}{1 + \frac{\frac{1}{2}}{\frac{3}{2}}} = \frac{\frac{1}{3}}{1 + \frac{2}{3}} = \frac{\frac{1}{3}}{\frac{5}{3}} = \frac{3}{5 \cdot 5} = \frac{1}{5} \quad \checkmark$$

$$\text{iii)} \frac{5 + \frac{3}{1 + \frac{2}{5}}}{1 + \frac{4 - \frac{2}{3}}{5}} = \frac{5 + \frac{3}{\frac{7}{5}}}{1 + \frac{\frac{10}{3}}{5}} = \frac{5 + \frac{15}{7}}{1 + \frac{2}{3}} = \frac{\frac{50}{7}}{\frac{5}{3}} =$$

$$= \frac{30}{7} \quad \checkmark$$

(3)

$$\begin{aligned}
 \text{i) } & 3(2a - 3b) - 4(-3a + 2(a + 2b - 1)) = \\
 & = 6a - 9b - 4(-3a + 2a + 4b - 2) = 6a - 9b - 4(-a + 4b - 2) \\
 & = 6a - 9b + 4a - 16b + 8 = 10a - 25b + 8 \\
 & = -5 \cdot 0.25 + 8 = 2.75 \quad \checkmark
 \end{aligned}$$

$$\begin{aligned}
 \text{ii) } & -2(-2b + a(-2b + 1)) + 3(-2(a + b) + (-2a - 4b + 3)) = \\
 & = -2(-2b - 2ab + a) + 3(-2a - 2b - 2a - 4b + 3) = \\
 & = 4b + 4ab - 2a - 6a - 6b - 6a - 12b + 9 = \\
 & = -14a - 14b + 4ab + 9 = -14(a + b) + 4 \cdot a \cdot b + 9 = \\
 & = +14 \cdot 0.49 + 4 \cdot 0.005 + 9 = 19,84
 \end{aligned}$$

(4)

$$\begin{aligned}
 & -3(2(1-3x) - 3(-3(2x-1) - 4(3x-5) - 2x+1)) = \\
 & = -3(2-6x - 3(-6x+3 - 12x+20 - 2x+1)) = \\
 & = -3(2-6x - 3(-20x+24)) = -3(2-6x + 60x - 72) = \\
 & = -3(-54x - 70) = -162x + 210 \quad \checkmark
 \end{aligned}$$

$$\textcircled{5} \quad \frac{1}{a} = \frac{\frac{1+x}{y}}{5x - 3(2y-2x)} = \frac{\frac{y+x}{y}}{5x-6y+6x} = \frac{\frac{y+x}{y}}{\underbrace{11x-6y}_1} =$$

$$= \frac{y+x}{y \cdot (11x-6y)} = \frac{y+x}{11xy-6y^2} =$$

$$\textcircled{6} \quad \text{i) } x \in \mathbb{R} \setminus \{0\} \quad \text{ii) } x \in \mathbb{R} \setminus \{1, -5\} \quad \text{iii) } x \in \mathbb{R} \setminus \left\{0, \frac{2}{3}\right\}$$

$$\textcircled{7} \quad \text{i) } a \in \mathbb{R} \quad , \quad \text{ii) } a \in \mathbb{R} \setminus \left\{0, -\frac{27}{5}\right\} , \quad \text{iii) } a \in \mathbb{R} \setminus \left\{-\frac{1}{2}, \frac{1}{2}\right\}$$

$$\textcircled{8} \quad x = -x \Leftrightarrow 2x = 0 \Leftrightarrow \underline{x = 0}$$

$$\textcircled{9} \quad x = \frac{1}{x} \Leftrightarrow x^2 = 1 \Leftrightarrow \underline{x = 1 \text{ in } x = -1}$$

⑩ Διαιρέσιν +ε ως $x-1$ ωστούσια οι.

⑪ Διαιρέσιν ων $(a-b)$ δων λογικά η είναι 0.

⑫ $\frac{a}{b} = \frac{2a - 5\gamma}{2b - 5\delta} \Leftrightarrow 2ab - 5a\delta = 2a\delta - 5\gamma b \Leftrightarrow$

$$a\delta = b\gamma \Leftrightarrow \frac{a}{b} = \frac{\gamma}{\delta} \quad \text{δων λογικά}$$

⑬ $\frac{a}{b} = \frac{a^2 + \gamma^2}{ab + \gamma\delta} \Leftrightarrow a^2b + a\gamma\delta = a^2\delta + b\gamma^2 \Leftrightarrow$

$$\Leftrightarrow a\delta = b\gamma \Leftrightarrow \frac{a}{b} = \frac{\gamma}{\delta} \quad \text{δων λογικά.}$$

⑭ Εγω x, y, z οι αποθετούμενοι.

Τότε $x+y+z=27$
 $\frac{x}{2} = \frac{y}{3} = \frac{z}{4} = \frac{x+y+z}{2+3+4} = \frac{27}{9} = 3$

Apa $\frac{x}{2} = 3 \Rightarrow \boxed{x=6}$
 $\boxed{y=9}, \boxed{z=12}$

(15)

$$x+y+z = 180$$

$$\frac{x}{1} = \frac{y}{3} = \frac{z}{5} = \frac{x+y+z}{9} = \frac{180}{9} = 20$$

Apa $x=20, y=60, z=100.$

(16)

$$\frac{a}{\delta} = \left(\frac{\beta}{\gamma}\right)^3 \Leftrightarrow \frac{a}{\delta} = \frac{\beta^3}{\gamma^3} \Leftrightarrow \frac{a \cdot \gamma^3}{\delta} = \beta^3 \cdot \underset{\downarrow}{\delta}$$

daher

$$\frac{a}{\delta} = \frac{\beta}{\gamma} \Leftrightarrow \beta^2 = a\gamma$$

$$\frac{\beta}{\gamma} = \frac{x}{\delta} \Leftrightarrow \gamma^2 = \beta\delta$$

$$\text{Apa } a \cdot \gamma \cdot \gamma^2 = \beta \cdot \beta^2 \cdot \delta \Leftrightarrow$$

$$a \cdot \gamma \cdot \beta \cdot \delta = \beta \cdot a \cdot \gamma \cdot \delta \text{ dau 1x ist -}$$

(17)

$$\frac{x}{y} = \frac{3}{4} \Leftrightarrow \left(x = \frac{3}{4}y \right)$$

$$\text{Apa } \frac{7x - 4y}{3x + y} = \frac{7 \cdot \frac{3}{4}y - 4y}{3 \cdot \frac{3}{4}y + y} = \frac{\frac{21}{4}y - \frac{16}{4}y}{\frac{9}{4}y + \frac{4}{4}y} = \\ = \frac{\frac{5y}{4}}{\frac{13y}{4}} = \frac{5y}{13y} = \frac{5}{13}.$$

(18)

$$\text{i) } \left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$\text{ii) } (-2)^3 \cdot (-0.5)^{-2} = -2^3 \cdot \left(\frac{1}{2}\right)^{-2} = \\ = -2^3 \cdot 2^2 = -2^5$$

$$\text{iii) } \left[\left(-\frac{1}{2}\right)^{-3} \cdot \left(\frac{1}{10}\right)^{-3} \right] : (-10)^2 = \left(-2^3 \cdot 10^3 \right) \cdot \frac{1}{10^2} =$$

$$= -\frac{-2^3 \cdot 10^3}{10^2} = -\underbrace{(2^3 \cdot 10)}_{=} = -80$$

(19)

$$\begin{aligned} A &= \left((x^2 y^{-1})^2 x^{-1} (y^3)^{-2} \right)^{-1} : \left(\frac{y^{-2}}{x^4} \right)^{-3} = \\ &= \left(x^4 y^{-2} x^{-1} y^{-6} \right)^{-1} \cdot \left(\frac{x^4}{y^{-2}} \right)^{-3} = \left(x^3 y^{-8} \right)^{-1} \cdot \left(\frac{y^{-2}}{x^4} \right)^3 \\ &= x^{-3} y^8 \cdot \frac{y^{-6}}{x^{12}} = \frac{y^2}{x^{15}} = \frac{10^{-2}}{10^{-30}} = \underline{\underline{10^{28}}} \end{aligned}$$

(20)

$$\begin{aligned} i) \quad &\left(\frac{3}{4} \right)^{25} : \left(\frac{4}{3} \right)^{-22} = \left(\frac{3}{4} \right)^{25} : \left(\frac{3}{4} \right)^{22} = \left(\frac{3}{4} \right)^{25} \cdot \left(\frac{4}{3} \right)^{22} = \\ &= \left(\frac{3}{4} \right)^3 \cdot \left(\frac{3}{4} \right)^{22} \cdot \left(\frac{4}{3} \right)^{22} = \left(\frac{3}{4} \right)^3 \cdot 1^{22} \end{aligned}$$

$$\text{ii)} \quad \left(\left(\frac{1}{2} \right)^{-3} \cdot \frac{1}{2^5} \right)^{-16} \cdot \left(\left(\frac{2^6}{10} \right)^{-1} : \left(\frac{1}{2^4} \right)^{-2} \right)^{-3} =$$

$$= \left(2^3 \cdot \frac{1}{2^5} \right)^{-16} \cdot \left(\frac{10}{2^6} : 2^8 \right)^{-3} = \left(\frac{1}{2^2} \right)^{-16} \cdot \left(\frac{10}{2^8 \cdot 2^6} \right)^{-3}$$

$$= 2^{32} \cdot \frac{10^{-3}}{2^{42}} = \frac{10^{-3}}{2^{10}} = \frac{1}{2^{10} \cdot 10^3}$$

(21)

$$\begin{aligned}
 & (x^3y^{-1})^2 : (x^{-1}y \cdot (x^3y^{-3})^{-1})^{-2} = x^6y^{-2} : (x^{-1}y x^{-3}y^3)^{-2} \\
 &= x^6y^{-2} : (x^{-4}y^4)^{-2} = x^6y^{-2} : (x^8y^{-8}) = \\
 &= \frac{x^6y^{-2}}{x^8y^{-8}} = \frac{y^6}{x^2} = \frac{\left[\left(-\frac{1}{10}\right)^{-2}\right]^6}{(10^{-3})^2} = \frac{\left(\frac{1}{10}\right)^{-12}}{10^{-6}} = \\
 &= \frac{10^{12}}{10^{-6}} = \underline{10^{18}}
 \end{aligned}$$

(22)

$$\begin{aligned}
 i) \quad & \left(\frac{x^2}{2y}\right)^5 \cdot \left(\frac{4y}{x}\right)^6 = \frac{x^{10}}{2^5 y^5} \cdot \frac{4^6 y^6}{x^6} = \frac{x^4 \cdot y \cdot 2^1}{2^5} \\
 &= x^4 y \cdot 2^7
 \end{aligned}$$

$$ii) \quad \frac{3a^2b^{-1}}{2ab^3} : \frac{2ab^{-2}}{3a^3b} = \frac{3a^2b^{-1}}{2ab^3} \cdot \frac{3a^3b}{2ab^{-2}} = \frac{9a^3}{4b}$$

(23)

$$\begin{aligned}
 & \left(\frac{a^2bc^3}{a^{-1}b^2c^4} \cdot \frac{(a^{-1}b c^{-2})^2}{(bc^{-2})^{-3}} \right) : (a^3b^{-4}c^2)^{-2} = \\
 & \frac{a^3}{bc} \cdot \frac{a^{-2}b^2c^{-4}}{b^{-3}c^{+6}} : (a^{-6}b^{+2}c^{-4}) = \frac{ab^4}{c^{11}} \cdot \frac{1}{a^{-6}b^2c^{-4}} \\
 &= a^7 b^2 \cdot c^{-7} = (5^{-2})^7 \cdot \left(\frac{1}{2 \cdot 5}\right)^2 \cdot \left(\frac{1}{5}\right)^7 = 5^{-14} \cdot \frac{1}{2^2 5^2} \cdot 5^7 \\
 &= 5^{-3} \cdot 2^{-2}
 \end{aligned}$$

(24)

$$\begin{aligned}
 \text{(i)} \quad & 5x(x-1)^2 - 3x(2x-3)^2 + (x-1)^3 - (2x+)^3 + 2x(4x-3)(4x+5) = \\
 & = 5x(x^2-2x+1) - 3x(4x^2-12x+9) + (x^3-3x^2+3x-1) \\
 & \quad - (8x^3+12x^2+6x+1) + 2x(16x^2-9) = \\
 & = 5x^3-10x^2+5x - 12x^3+36x^2-27x + x^3-3x^2+3x-1 \\
 & \quad - 8x^3-12x^2-6x-1 + 32x^3-18x = \\
 & = +18x^3+11x^2-43x-2
 \end{aligned}$$

Options

$$\text{ii)} \quad = \dots = -x^3 - 7x^2 - 5x + 5$$

$$\text{iii)} \quad = \dots = -8a^3 + 19a^2b + 7ab^2 + 8b^3$$

$$\text{(25)} \quad = \dots = -17 + 16x - 14x^2 + 6x^3 - x^4$$

$$\begin{aligned}
 \text{(26)} \quad & (a+b+c)^2 - (a-b+c)^2 + (a+b-c)^2 - (a-b-c)^2 = \\
 & a^2 + b^2 + c^2 + 2ab + 2ac + 2bc - (a^2 + b^2 + c^2 - 2ab + 2ac - 2bc) \\
 & + (a^2 + b^2 + c^2 + 2ab - 2ac - 2bc) - (a^2 + b^2 + c^2 - 2ab - 2ac + 2bc) \\
 & a^2 + b^2 + c^2 + 2ab + 2ac + 2bc - a^2 - b^2 + c^2 + 2ab - 2ac + 2bc \\
 & + a^2 + b^2 + c^2 + 2ab - 2ac - 2bc - a^2 - b^2 - c^2 + 2ab + 2ac - 2bc = \\
 & = \cancel{8ab}
 \end{aligned}$$

(28)

$$(a^2 + b^2)^2 + 4ab(a^2 - b^2) = (a^2 - b^2 + 2ab)^2 \Leftrightarrow$$

$$a^4 + b^4 + 2a^2b^2 + 4a^3b - 4ab^3 = a^4 + b^4 + 4a^2b^2 - 2a^2b^2 + 4a^3b - 4ab^3 \Leftrightarrow$$

$$2a^2b^2 = 2a^2b^2 \text{ also true.}$$

(27)

$$(a^2 - b^2)^2 + (2ab)^2 = (a^2 + b^2)^2 \Leftrightarrow$$

$$a^4 + b^4 - 2a^2b^2 + 4a^2b^2 = a^4 + b^4 + 2a^2b^2 \Leftrightarrow$$

$$2a^2b^2 = 2a^2b^2 \text{ also true.}$$

(29)

$$\text{i)} a^2 + b^2 = (a+b)^2 - 2ab = 5^2 - 2 \cdot 4 = 25 - 8 = 17$$

$$\text{ii)} a^4 + b^4 = (a^2 + b^2)^2 - 2a^2b^2 = 17^2 - 2 \cdot 4^2 = 257$$

$$\text{iii)} a^3 + b^3 = (a+b) \cdot (a^2 - ab + b^2) = 5 \cdot (17 - 8) = 45$$

$$\text{iv)} a^6 + b^6 = (a^2)^3 + (b^2)^3 = (a^2 + b^2) \cdot (a^4 - a^2b^2 + b^4) =$$

$$= (17) \cdot (257 - 4^2) = 4097$$

(30)

$$2(x^2 + y^2) = (x+y)^2 \Leftrightarrow 2x^2 + 2y^2 = x^2 + y^2 + 2xy \Leftrightarrow$$

$$2x^2 + 2y^2 - x^2 - y^2 - 2xy = 0 \Leftrightarrow x^2 + y^2 - 2xy = 0 \Leftrightarrow$$

$$(x-y)^2 = 0 \Leftrightarrow x = y$$

(31)

$$1 + \frac{a+b}{a} + \frac{a+b}{b} = 5 \quad \Leftrightarrow \quad \begin{array}{l} ab \neq 0 \\ ab + ab \cdot \frac{a+b}{a} + ab \cdot \frac{a+b}{b} = 5ab \end{array}$$

$$\Leftrightarrow ab + b(a+b) + a(a+b) = 5ab \Leftrightarrow$$

$$ab + ab + b^2 + a^2 + ab = 5ab \Leftrightarrow a^2 + b^2 - 2ab = 0 \Leftrightarrow$$

$$(a-b)^2 = 0 \Leftrightarrow \underline{\underline{a=b}}$$

(32)

$$a - \frac{1}{a} = 5 \Leftrightarrow \left(a - \frac{1}{a}\right)^2 = 25 \Leftrightarrow a^2 + \frac{1}{a^2} - 2 \cdot a \cdot \frac{1}{a} = 25 \Leftrightarrow$$

$$a^2 + \frac{1}{a^2} - 2 = 25 \Leftrightarrow \boxed{a^2 + \frac{1}{a^2} = 23}$$

(33)

$$\text{i)} \quad (a+b)^2 - (a-b)^2 = a^2 + b^2 + 2ab - a^2 - b^2 + 2ab = \\ = \underline{\underline{4ab}}$$

$$\text{ii)} \quad \text{Satz zu} \quad a \cdot b = \frac{(a+b)^2 - (a-b)^2}{4} = \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2$$

$$\text{iii)} \quad 17 \cdot 13 = \left(\frac{17+13}{2}\right)^2 - \left(\frac{17-13}{2}\right)^2 = 15^2 - 2^2$$

(34)

$$a^2 + b^2 + 2 = 2(a+b) \Leftrightarrow a^2 + b^2 + 2 = 2a + 2b \Leftrightarrow$$

$$a^2 + b^2 + 2 - 2a - 2b = 0 \Leftrightarrow a^2 - 2a + 1 + b^2 - 2b + 1 = 0 \Leftrightarrow \\ (\text{weil } 2 = 1+1)$$

$$(a-1)^2 + (b-1)^2 = 0 \Leftrightarrow a=1 \text{ und } b=1.$$

(35)

$$x^3(y+1) - y^3(x+1) = x-y \Leftrightarrow x^3(y+1) - x - y^3(x+1) + y = 0$$

$$\Leftrightarrow \underline{x^3y} + \underline{x^3} - \underline{x} - \underline{y^3x} - \underline{y^3} + \underline{y} = 0 \Leftrightarrow$$

$$xy(x^2 - y^2) + (x-y)(x^2 + xy + y^2) + (y-x) = 0 \Leftrightarrow$$

$$xy(x-y)(x+y) + (x-y)(x^2 + xy + y^2) + (y-x) \stackrel{x+y=1}{=} 0 \Leftrightarrow$$

$$xy(x-y) + (x-y)(x^2 + xy + y^2) - (x-y) = 0 \Leftrightarrow$$

$$(x-y) \left[\underline{xy} + \underline{x^2} + \underline{xy} + \underline{y^2} - 1 \right] = 0 \Leftrightarrow$$

$$(x-y) \left[(x+y)^2 - 1 \right] = 0 \Leftrightarrow (x-y) \cdot 0 = 0 \quad \text{1c x u.}$$

(36)

$$(a+b) \left(\frac{1}{a} + \frac{1}{b} \right) = 4 \Leftrightarrow a \cdot \frac{1}{a} + a \cdot \frac{1}{b} + b \cdot \frac{1}{a} + b \cdot \frac{1}{b} = 4 \Leftrightarrow$$

$$1 + \frac{a}{b} + \frac{b}{a} + 1 = 4 \Leftrightarrow \frac{a}{b} + \frac{b}{a} = 2 \stackrel{ab \neq 0}{\Leftrightarrow}$$

$$ab \cdot \frac{a}{b} + ab \cdot \frac{b}{a} = 2ab \Leftrightarrow a^2 + b^2 - 2ab = 0 \Leftrightarrow (a-b)^2 = 0 \Leftrightarrow$$

$$\underline{\underline{a=b}}$$

(37)

$$\text{Für } a \neq b \quad a^3 + b^3 = (a+b) \cdot (a^2 - ab + b^2)$$

$$\text{Apa} \quad a^3 + b^3 = a^2 + b^2 - ab \quad (\text{da } a+b=1)$$

$$\text{Einsetzen} \quad a^2 + b^2 = (a+b)^2 - 2ab = 1 - 2ab$$

$$\text{Ergebnis} \quad a^3 + b^3 = 1 - 2ab - ab = 1 - 3ab$$

Apa

$$\underline{a^3 + b^3 + 3ab} = 1 - 3ab + 3ab = 1.$$

(38)

$$(a-b)^3 + (b-c)^3 + (c-a)^3 = 3(a-b)(b-c)(c-a) \quad (=)$$

$$a^3 - 3a^2b + 3ab^2 + b^3 - 3b^2c + 3bc^2 + c^3 + c^3 - 3c^2a + 3ca^2 + a^3 = \\ = 3(ab - ac - b^2 + bc)(c-a) \quad (\neq)$$

$$3(-a^2b + ab^2 - b^2c + bc^2 - c^2a + ca^2) = 3\left(abc - a^2b - ac^2 + a^2c - b^2c + b^2a + bc^2 - abc\right)$$

Dann lösbar.

(39)

$$\begin{aligned}
 y &= (3a - 4a^3)^2 + (3b - 4b^3)^2 = \\
 &= 9a^2 - 24a^4 + 16a^6 + 9b^2 - 24b^4 + 16b^6 = \\
 &= 9(a^2 + b^2) + 16(a^6 + b^6) - 24(a^4 + b^4)
 \end{aligned}$$

Ophys $a^6 + b^6 = (a^2)^3 + (b^2)^3 = (a^2 + b^2) \cdot (a^4 - a^2b^2 + b^4)$
 uor $a^4 + b^4 = (a^2)^2 + (b^2)^2 = (a^2 + b^2)^2 - 2a^2b^2$

Endeivus, αρού $a^2 + b^2 = 1$

$$\begin{aligned}
 y &= 9 + 16 \cdot (a^4 + b^4 - a^2b^2) - 24 \left[1 - 2a^2b^2 \right] = \\
 &= 9 + 16 \cdot \left[1 - 2a^2b^2 - a^2b^2 \right] - 24 + 48a^2b^2 \\
 &= 9 + 16 - 48a^2b^2 - 24 + 48a^2b^2 = 1
 \end{aligned}$$

(40)

i) $7(3x+1)^2 - 28(x-2)^2 = 7(9x^2 + 6x + 1) - 28(x^2 - 4x + 4) =$
 $= 63x^2 + 42x + 7 - 28x^2 + 112x - 112 =$
 $= 35x^2 + 154x - 105 = 7(5x^2 + 22x - 15) = \dots =$
 $= 7(x+5)(5x-3)$ (ερετε πιστε σαμπινωγα)

ii) $(a-1)^3(a^2-4) + 4-a^2 = (a^2-4) \cdot [(a-1)^3 - 1^3] =$
 $= (a-2)(a+2) \left[(a-1) - 1 \right] \cdot \left[(a-1)^2 + a-1 + 1 \right] =$
 $= (a-2)(a+2) \cdot (a-2) \cdot (a^2 - 2a + 1 + a) =$
 $= (a-2)^2(a+2)(a^2 - a + 1)$

$$\text{iii) } \underbrace{x^2 - 2xy + y^2}_{= (x-y)^2} - x + y = (x-y)^2 - (x-y) = \\ = (x-y) \cdot (x-y-1)$$

$$\text{iv) } (a^2 + b^2 - c^2)^2 - 4a^2b^2 = (a^2 + b^2 - c^2 - 2ab)(a^2 + b^2 - c^2 + 2ab) = \\ = ((a-b)^2 - c^2) ((a+b)^2 - c^2) = (a-b-c)(a-b+c)(a+b-c)(a+b+c)$$

$$\text{v) } (2a-3b)(x-3y) + (3a-b)(-x+3y) = (2a-3b)(x-3y) - (3a-b)(x-3y) \\ = (x-3y)(2a-3b - 3a+b) = (x-3y)(-a+2b) = -(x-3y)(a+2b)$$

$$\text{vi) } \underbrace{x^3 + x^2 - x - 1}_{= x(x^2-1) + x^2-1} = x(x^2-1) + x^2-1 = (x^2-1) \cdot (x+1) = \\ = (x-1)(x+1)(x+1) = (x-1)(x+1)^2$$

$$\text{vii) } (2x-3)(x+1) + (3-2x)(2x+8) + 4x^2 - 9 = \\ = (2x-3)(x+1) - (2x-3)(2x+8) + (2x)^2 - 3^2 = \\ = (2x-3)(x+1) - (2x-3)(2x+8) + (2x-3)(2x+3) = \\ = (2x-3)(x+1 - 2x-8 - 2x-3) = (2x-3)(-3x-10)$$

$$\text{viii) } a^4b - a^4b^4 + b^3 - a^3 = ab(a^3 - b^3) - (a^3 - b^3) = \\ = (a^3 - b^3)(ab - 1) = (a-b)(a^2 + ab + b^2) \cdot (ab - 1)$$

$$④① \text{ i) } \frac{x^2+5x+6}{2x^2+4x} = \frac{(x+2)(x+3)}{2x(x+2)} = \frac{x+3}{2x}$$

$$\begin{aligned} \text{ii) } & \frac{a^5 - a^3 + a^2 - 1}{a^5 + a^2 - a - 1} = \frac{a^3(a^2 - 1) + (a^2 - 1)}{a^2(a+1) - (a+1)} = \\ & = \frac{(a^2 - 1)(a^3 + 1)}{(a+1)(a^2 - 1)} = \frac{(a+1)(a^2 - a + 1)}{a+1} = a^2 - a + 1 \end{aligned}$$

$$④② ax^2 + ax + a = bx^2 + bx + b \Leftrightarrow$$

$$\begin{aligned} & \underline{ax^2 + ax + a} - \underline{bx^2 - bx - b} = 0 \Leftrightarrow \\ & x^2(a-b) + x(a-b) + a-b = 0 \Leftrightarrow (a-b)(x^2+x+1) = 0 \Leftrightarrow \\ & a-b = 0 \Leftrightarrow \underline{\underline{a=b}} \end{aligned}$$

$$\left(x^2 + x + 1 \neq 0, \text{ apoi } \Delta < 0 \right)$$

$$\begin{aligned} ④③ & \left(\frac{a+b}{a-b} + \frac{a^2+b^2}{a^2-b^2} \right) : \left(\frac{a-b}{a+b} - \frac{a^3-b^3}{a^3+b^3} \right) = \\ & \left(\frac{\frac{ab}{a-b} + \frac{1}{a^2-b^2}}{\frac{a+b}{a-b} + \frac{a^2+b^2}{(a-b)(a+b)}} \right) : \left(\frac{\frac{a^2-ab+b^2}{a^2-b^2}}{\frac{a-b}{a+b} - \frac{a^3-b^3}{(a+b)(a^2-ab+b^2)}} \right) = \\ & \frac{(a+b)^2 + a^2 + b^2}{(a-b)(a+b)} : \frac{(a-b)(a^2-ab+b^2) - a^3 + b^3}{(a+b)(a^2-ab+b^2)} = \end{aligned}$$

$$\frac{2a^2 + 2b^2 + 2ab}{(a-b)(a+b)} \cdot \frac{(a+b)(a^2 - ab + b^2)}{(a-b)(a^2 - ab + b^2) + (b-a)(b^2 + ab + a^2)} =$$

$$= \frac{2(a^2 + b^2 + ab)(a^2 - ab + b^2)}{(a-b)^2 [a^3 - ab + b^2 - b^2 - ab + a^2]} = \frac{2(a^2 + ab + b^2)(a^2 - ab + b^2)}{-2(a-b)^2(ab)}$$

$$= \frac{(a^2 + ab + b^2)(a^2 - ab + b^2)}{-(a-b)^2 \cdot ab}$$

(44)

$$i) \frac{x^2+x}{x^2-1} = \frac{x(x+1)}{x(x-1)} = \frac{x+1}{x-1}$$

$$ii) \frac{4x - x^3}{x^2 - 4x + 4} = \frac{x(4-x^2)}{(x-2)^2} = \frac{x(2-x)(2+x)}{(x-2)^2} = \frac{x(2+x)}{2-x}$$

$$iii) \frac{\overbrace{a^2 + b^2 - c^2 + 2ab}^{\text{Difference of squares}}}{\overbrace{a^2 - b^2 + c^2 + 2ac}^{\text{Difference of squares}}} = \frac{(a+b)^2 - c^2}{(a+c)^2 - b^2} = \frac{(a+b+c)(a+b-c)}{(a+c+b)(a+c-b)} =$$

$$= \frac{a+b-c}{a+c-b}$$

$$iv) \frac{a^2 - b^2}{a-b} - \frac{a^3 - b^3}{a^2 - b^2} = \frac{(a-b)(a+b)}{(a-b)} - \frac{(a-b)(a^2 + ab + b^2)}{(a-b)(a+b)}$$

$$= \frac{a+b}{1} - \frac{a^2 + ab + b^2}{a+b} = \frac{(a+b)^2 - a^2 - ab - b^2}{(a+b)} =$$

$$= \frac{a^2 + b^2 + 2ab - a^2 - ab - b^2}{(a+b)} = \frac{ab}{a+b}$$

- (45) (Δ δίδωμα + ε ανταργύρι εις άριστο) Εγω ότι ο x είναι αριθμός τότε
Ως υπάρχει $x \in \mathbb{Z}$: $x = 2 \cdot k$. Εποτέρως
 $x^3 = (2 \cdot k)^3 \Rightarrow x^3 = 8k^3 \Rightarrow \underline{\underline{x^3 = 2 \cdot (4k^3)}}$
Άρα ο x^3 είναι αριθμός. Άριστο.

(46) Εγω $2v+1, 2v+3$ διαδοχικοί δημιουργοί είναι αριθμοί.
Αν διαφορά των υπών των θα είναι:

$$(2v+3)^3 - (2v-1)^3 = (2v+3-2v+1) \cdot \left[(2v+3)^2 + (2v+3)(2v+1) + (2v+1)^2 \right]$$

$$= 2 \cdot \left[(2v+3)^2 + (2v+3)(2v+1) + (2v+1)^2 \right]$$
 αριθμός αριθμός

$$= 2 \cdot [4v^2 + 12v + 9 + 4v^2 + 2v + 6v + 3 + 4v^2 + 4v + 1]$$

$$= 2 \cdot (12v^2 + 24v + 13) = 2 \cdot [12(v^2 + 2v) + 12 + 1]$$

$$= 2 \cdot [12 \cdot (v^2 + 2v + 1) + 1] = 2 \cdot [12(v+1)^2 + 1]$$

Ο αριθμός $12(v+1)^2 + 1 = 2 \cdot (6(v+1)^2) + 1$ είναι
δημιουργός. Εποτέρως δεν διαιρείται + ε ρο 2. Άρα
η διαφορά των υπών $(2v+3)^3 - (2v-1)^3$ δεν
διαιρείται διε ρο 4.

(47) Ε6ων $v, v+1, v+2, v+3$ 4 συστοχικοί ανέργοι.

To αποτέλεσμα είναι:

$$v+v+1+v+2+v+3 = 4v+6 = \underline{4(v+1)+2}.$$

O αριθμός αυτός είναι αριθμός, ο οποίος διαιρίζεται
με το 4 (το μετατόπισμα της συμβολής + είναι 4 είναι 2)

(48) Ε6ων $v-1, v, v+1, v+2$ 4 συστοχικοί ανέργοι.

Tοτε

$$\begin{aligned} (v-1)v(v+1)(v+2) + 1 &= \dots = v^4 + 2v^3 - \cancel{v^2} - 2v + 1 = \\ &= v^4 + v^2 + 1 - 2v^2 + 2v^3 - 2v \\ &= (v^2 + v + 1)^2 \end{aligned}$$

(49) $\frac{a}{1} \neq \frac{b}{2} = \frac{c}{4} = \frac{a+b+c}{7}$ ($a, b, c \in \mathbb{Z}$)

$$\frac{a}{1} = \frac{a+b+c}{7} \Rightarrow \frac{a+b+c = 7 \cdot a}{\hookrightarrow \text{no 2/6 so even 7}}$$

(50) Ερώτηση για δύο χιλιότερους απέριττους

$$(v+1)^2 - v^2 = v^2 + 2v + 1 - v^2 = \frac{2v+1}{\hookrightarrow \text{περιττούς}}$$

(51) Ερώτηση για δύο επαρχίους απέριττους

$$(2v+1)^2 = 4v^2 + 4v + 1 = 4(v^2 + v) + 1$$

Για το να μάρκων δύο διαδικασίες

α) Αν v αριθμός, δηλαδή $v = 2k$ τότε

$$v^2 + v = 4k^2 + 2k = 2(2k^2 + k)$$

Επομένως

$$(2v+1)^2 = 4 \cdot 2(2k^2 + k) + 1 = \underline{\underline{8 \cdot (2k^2 + k) + 1}}_{\text{οκ}}$$

β) Αν v μεριτής, δηλαδή $v = 2k+1$ τότε

$$\begin{aligned} v^2 + v &= (2k+1)^2 + 2k+1 = 4k^2 + 4k + 1 + 2k + 1 = \\ &= 4k^2 + 6k + 2 = 2 \cdot (2k^2 + 3k + 1) \end{aligned}$$

Επομένως

$$(2v+1)^2 = 4 \cdot 2 \cdot (2k^2 + 3k + 1) + 1 = 8 \cdot (2k^2 + 3k + 1)$$