

**Title:** *Normalizers and Approximate Units for Inclusions of  $C^*$ -Algebras*

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**Abstract:** Consider *inclusions*, which are pairs of  $C^*$ -algebras  $(C, D)$  with  $D$  an abelian subalgebra of  $C$ . An element  $v \in C$  *normalizes*  $D$  if  $v^*Dv \cup vDv^* \subseteq D$ . The inclusion  $(C, D)$  is *regular* when the linear span of the normalizers is dense in  $C$  and is *singular* when every normalizer belongs to  $D$ .

I will prove a commutation result for Hermitian normalizers, then discuss some consequences of this result related to familiar constructions. Sample consequence: when  $D$  is a regular MASA in  $C$ , every approximate unit for  $D$  is an approximate unit for  $C$ ; this leads to simplification of the notions of Cartan MASA and  $C^*$ -diagonal in the non-unital setting.

The inclusion  $(C, D)$  is *intermediate* to the regular MASA inclusion  $(B, D)$  if  $D \subseteq C \subseteq B$ . I will give examples showing some singular MASA inclusions are intermediate to regular MASA inclusions, but others are not, and will discuss the fact that when  $\mathcal{H}$  is a separable, infinite dimensional Hilbert space, no MASA inclusion of the form  $(\mathcal{B}(\mathcal{H}), D)$  is intermediate to a regular MASA inclusion.

(Based on [arXiv:2109.00856](#))