Semigroup C*-algebras and their K-theory

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- Semigroup C*-algebras

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- Examples

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- The underlying dynamics

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- K-theory for semigroup C*-algebras

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- K-theory for semigroup C*-algebras
- Examples and applications
- A more general K-theory formula

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- Non-self-adjoint versions:

$$A_\lambda(P) = \overline{\mathrm{span}}(\{\lambda_p \colon p \in P\}) \subseteq \mathcal{L}(\ell^2 P).$$

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  $P = \mathbb{N} * \mathbb{N}$, $0 \to \mathcal{K} \to C^*_\lambda(\mathbb{N} * \mathbb{N}) \to \mathcal{O}_2 \to 0$.

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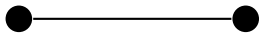

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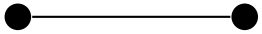

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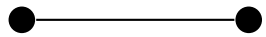

Theorem (Eilers-L-Ruiz): $C^*_\lambda(A_\Gamma^+) \cong C^*_\lambda(A_\Lambda^+)$ if and only if

1. $t(\Gamma) = t(\Lambda)$
2. $N_k(\Gamma) + N_{-k}(\Gamma) = N_k(\Lambda) + N_{-k}(\Lambda)$ for all $k \in \mathbb{Z}$
3. $N_0(\Gamma) > 0$, or $\sum_{k>0} N_{-k}(\Gamma) \equiv \sum_{k>0} N_{-k}(\Lambda) \mod 2$

Here t and $N_\bullet$ are invariants of graphs.

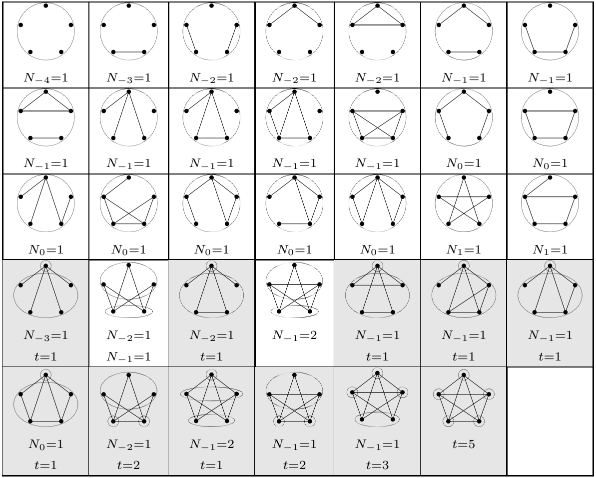

Figure 1: Invariants for all graphs with 5 vertices. Any quantity not mentioned equals zero.

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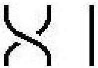
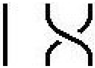


σ_1



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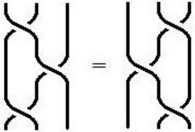

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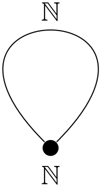
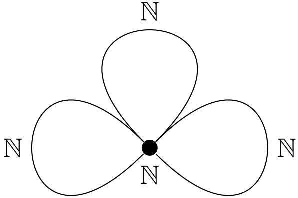
  Graphs of monoids [Chen-L]:





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  such that their C*-algebras $C^*_\lambda(P_1)$ and $C^*_\lambda(P_2)$ are nuclear,
  then $C^*_\lambda(P_1) \cong C^*_\lambda(P_2)$ if and only if $|\mathcal{S}_1| = |\mathcal{S}_2|$.

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P: semigroup		
$P \subseteq G$		
P sat. independence		
G sat. Baum-Connes		
$K_*(C^*_\lambda(P)) \cong \bigoplus_{[X] \in \mathcal{J}^\times/_\sim} K_*(C^*_\lambda(G_X))$		

A more general K-theory formula

P: semigroup	S: inverse semigroup	
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P: semigroup	S: inverse semigroup	$G \curvearrowright X$: partial dynamical system
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The End

Thank you very much!