Functional Analysis and Operator Algebras Seminar
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C*-ALGEBRAS GENERATED BY ISOMETRIES: 60 YEARS AND COUNTING

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The first talk will be a (necessarily biased and partial) survey of the history of C*-algebras generated by isometries on Hilbert space. I will begin by recalling classical theorems of Coburn, Douglas, and Cuntz from the 1960's and 1970's and then discuss their proofs. Douglas' and Cuntz's approaches already indicate, in an implicit way, that semigroup crossed products play a central role. This was not formalized until the late 1980's and early 1990's when Murphy, Stacey, Nica, and then Raeburn and I developed an explicit semigroup crossed product approach for Toeplitz algebras, focusing on a covariance condition that works quite well for quasi-lattice ordered groups. I will elaborate a bit on this approach and show how it works in a few examples. I will finish by discussing briefly the semigroup C*-algebra $C_s^*(P)$ introduced by Xin Li in the 2010's using constructible right ideals to generalize Nica's covariance condition, and will finish by giving some non quasi-lattice ordered examples from number theory.

The second talk will be on my joint work with Sehnem from the 2020's about a universal Toeplitz algebra $\mathcal{T}_u(P)$ defined via generators and relations whenever P is a submonoid of a group G. The C*-algebra $\mathcal{T}_u(P)$ coincides with Xin Li's $C_s^*(P)$ when the semigroup satisfies his independence condition but behaves as expected also when independence fails; for example, it is isomorphic to the C*-algebra of the left regular representation when the group G is amenable and also in many nonamenable situations. I will give a characterization of faithful representations and a uniqueness theorem for these universal Toeplitz algebras, which are new results even for right LCM monoids. Time permitting I will also discuss how Sehnem's covariance algebra of a product system leads to a full boundary quotient of $\mathcal{T}_u(P)$, generalizing the boundary relations of quasi-lattice orders introduced by Crisp and myself in the 2000's.