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Stable isomorphism of dual operator spaces. (English summary)
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This paper is a continuation of the first two authors' study of the stable isomorphism of dual operator algebras [see Math. Ann. **341** (2008), no. 1, 99–112; MR2377471 (2009a:46111)]. This paper is concerned instead with dual operator spaces, on the face of it a much larger class of objects, for which it is surprising that such strong results hold.

A dual operator space X is, concretely, a weak* closed subspace of $B(H, K)$ for some Hilbert spaces H and K; such a Banach space carries matrix norms, and abstractly we can think of X as being the dual of some operator space, equipped with its natural matrix norms. A ternary ring of operators (TRO) is a subspace $M \subseteq B(H, K)$ with $MM^*M \subseteq M$. We say that dual operator spaces $X \subseteq B(K_1, K_2)$ and $Y \subseteq B(H_1, H_2)$ are TRO-equivalent if there exist TROs $M_i \subseteq B(H_i, K_i)$ with X being the weak*-closed linear span of $M_2 Y M_1^*$ and Y being the weak*-closed linear span of $M_2^* X M_1$. For abstract dual operator spaces X and Y, we say that X and Y are Δ-equivalent if there exist representations $X \subseteq B(K_1, K_2)$ and $Y \subseteq B(H_1, H_2)$ which are TRO-equivalent. Finally, X and Y are stably isomorphic if the matrix spaces $M_I(X)$ and $M_I(Y)$ are weak*-completely isometric, for some index set I.

These are the obvious generalizations to dual operator spaces of the definitions for unital dual operator algebras, for which it is known that Δ-equivalence and stable isomorphism are the same [op. cit.]. The main, surprising, result of this paper is that Δ-equivalence and stable isomorphism are the same for dual operator spaces. The main idea is to associate natural algebras to X and Y, and then use the corresponding result for dual operator algebras: the use of multiplier algebras is essential for this.

An application of this is that, for example, if unital dual operator algebras A and B are Δ-equivalent, then there is a dual operator space X such that A is the space of left multipliers of X, and B is the space of right multipliers of X. Furthermore, the noncommutative Banach-Stone theorem shows that the various notions of stably isomorphic and Δ-equivalence, when restricted to unital dual operator algebras, all agree. The paper makes some applications to H^∞ function algebras, and to commutative subspace lattice algebras. The paper is nicely written, and relatively self-contained.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.