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MR2377471 (2009a:46111) 46L07 (47L35 47L45)
Eleftherakis, G. K. (GR-UATH); **Paulsen, V. I.** (1-HST)
Stably isomorphic dual operator algebras. (English summary)
Math. Ann. **341** (2008), *no. 1*, 99–112.

M. A. Rieffel introduced a notion of Morita equivalence for von Neumann algebras in [J. Pure Appl. Algebra **5** (1974), 51–96; MR0367670 (51 #3912)]. We can take as a definition that von Neumann algebras M and N are Morita equivalent if they are stably isomorphic. That is, for some index set I, the matrix algebras $M_I(M)$ and $M_I(N)$ are isomorphic as von Neumann algebras.

The paper under review is part of an ongoing program to generalise such notions to dual operator algebras. A dual operator algebra is, concretely, a weak operator closed (but possibly non-self-adjoint) subalgebra of $B(H)$, the operators on a Hilbert space H. Alternatively, such algebras can be characterised abstractly through the use of operator spaces and the Haagerup tensor product [see C. Le Merdy, Amer. J. Math. **121** (1999), no. 1, 55–63; MR1704997 (2001f:46086)].

Thus, dual operator algebras A and B are stably isomorphic if $M_I(A)$ and $M_I(B)$ are weak*-completely isometrically isomorphic, for some index set I. G. K. Eleftherakis [J. Pure Appl. Algebra **212** (2008), no. 5, 1060–1071; MR2387585 (2008m:47098)] defined two other equivalence relations. Suppose that A and B are subalgebras of $B(H)$ and $B(K)$ respectively. A ternary ring of operators (TRO) is a subspace $\mathcal{M} \subseteq B(H,K)$ with $\mathcal{M}\mathcal{M}^*\mathcal{M} \subseteq \mathcal{M}$. Then A and B are TRO equivalent if and only if there exists a TRO $\mathcal{M}$ with A being the weak*-closure of the span of $\mathcal{M}^*B\mathcal{M}$, and B being the weak*-closure of the span of $\mathcal{M}B\mathcal{M}^*$.

Given an abstract dual operator algebra A, a normal representation of A is a completely contractive, weak*-continuous homomorphism $\alpha\colon A \to B(H)$. We say that A and B are Δ-equivalent if there exist completely isometric normal representations α and β, of A and B respectively, such that $\alpha(A)$ and $\beta(B)$ are TRO equivalent. Eleftherakis [op. cit.] showed that Δ-equivalence can also be formulated in a category-theoretic sense.

The main result of the paper is to show that A and B are stably isomorphic if and only if they are Δ-equivalent. In the introduction, it is quickly shown that stably isomorphic algebras are Δ-equivalent; the hard work lies in showing the converse.

The main technical tool is to construct bimodules X and Y (that is, $X \subseteq B(K,H)$ with $AXB \subseteq X$ and $Y \subseteq B(H,K)$ with $BYA \subseteq Y$) such that

$$A \cong X \otimes_B^{\sigma h} Y, \quad B \cong Y \otimes_A^{\sigma h} X,$$

where these are balanced versions of the normal Haagerup tensor product.

The paper finishes with a quick application to commutative subspace lattice (CSL) algebras. The paper is well written, and pleasingly self-contained, with all the major terminology defined and explained.

Reviewed by *Matthew D. Daws*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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