MR2669424 (2011i:47107) 47L30 (16D90 46M15 47L35 47L45 47L55)
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Morita type equivalences and reflexive algebras. (English summary)
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A dual operator algebra is an abstract operator algebra which is the operator dual of an operator space. For every dual operator algebra $\mathcal{A}$ there exist a Hilbert space H and an algebraic homomorphism $\alpha\colon \mathcal{A}\to B(H)$ which is a complete isometry and a w*-continuous map. In particular, W*-algebras are dual operator algebras. In [G. K. Eleftherakis, "TRO equivalent algebras", preprint, arXiv:math/0607488, Houston J. Math., to appear; J. Pure Appl. Algebra **212** (2008), no. 5, 1060–1071; MR2387585 (2008m:47098); corrigendum, J. Pure Appl. Algebra **212** (2008), no. 11, 2581–2582; MR2440270 (2009k:47239)] the author obtained a generalization of Rieffel's concept of Morita equivalence of W*-algebras to the class of (not necessarily self-adjoint) unital dual operator algebras. Two unital dual operator algebras are Δ-equivalent if there is an equivalence functor between their categories of normal representations which not only preserves intertwiners of representations of the algebras, but also preserves intertwiners of their restrictions to the diagonals (where $\Delta(\mathcal{A})=\mathcal{A}\cap\mathcal{A}^*$). For two w*-closed algebras $\mathcal{A}$ and $\mathcal{B}$ acting on Hilbert spaces H_1 and H_2, respectively, the existence of a ternary ring of operators (TRO) $\mathcal{M}\subset B(H_1,H_2)$ such that $\mathcal{A}=[\mathcal{M}^*\mathcal{B}\mathcal{M}]^{-w^*}$ and $\mathcal{B}=[\mathcal{M}\mathcal{A}\mathcal{M}^*]^{-w^*}$ means that $\mathcal{A}$ and $\mathcal{B}$ are TRO equivalent. One central result of the previous publications is the following: Two unital dual operator algebras $\mathcal{A}$, $\mathcal{B}$ are Δ-equivalent if and only if they admit completely isometric normal representations α, β on Hilbert spaces such that the algebras $\alpha(\mathcal{A})$ and $\beta(\mathcal{B})$ are TRO equivalent.

The investigations of the paper under review focus on properties of any functor $\mathcal{F}$ which implements the described equivalence. The proofs are constructive and show a lot of details of the structures under consideration and of their interrelations.

For unital dual operator algebras $\mathcal{A}$ and $\mathcal{B}$ a functor $\mathcal{G}$ between the respective categories of normal representations which preserves intertwiners is said to be completely isometric (resp. normal) if for every pair of objects H_1, H_2 the map $\mathcal{G}\colon \mathrm{Hom}_{\mathcal{A}}(H_1,H_2)\to\mathrm{Hom}_{\mathcal{B}}(\mathcal{G}(H_1),\mathcal{G}(H_2))$ is a complete isometry (resp. w*-continuous). A functor $\mathcal{G}$ respects isometries if whenever (H,α) represents an object-homomorphism pair and $\alpha\colon\mathcal{A}\to B(H)$ is a complete isometry, the corresponding map $\mathcal{G}(\alpha)\colon B\to B(\mathcal{G}(H))$ is a complete isometry, too. One of the central results is that functors $\mathcal{F}$ respect isometries.

If $\mathcal{A}$ is any subalgebra of $B(H)$ for some Hilbert space H the set of projections L of $B(H)$ defined by $\mathrm{Lat}(\mathcal{A})=\{L\in pr(B(H))\colon L^\perp\mathcal{A}L=0\}$ is a lattice. A functor $\mathcal{G}$ is called a lattice respecting functor if for every object-homomorphism pair (H,α) the set identity $\mathcal{G}(\mathrm{Lat}(\alpha(\mathcal{A})))=\mathrm{Lat}(\beta(\mathcal{B}))$ holds, where $(\mathcal{G}(H),\beta)$ is the corresponding object in the image category of $\mathcal{G}$. A second central result is that functors $\mathfrak{F}$ are lattice respecting.

For every subalgebra $\mathcal{A}$ of some $B(H)$ set $\mathrm{Ref}(\mathcal{A})=\mathrm{Alg}(\mathrm{Lat}(\mathcal{A}))=\{X\in B(H)\colon L^\perp XL=0\text{ for any }L\in\mathrm{Lat}(\mathcal{A})\}$. An algebra $\mathcal{A}$ is reflexive if $\mathcal{A}=\mathrm{Ref}(\mathcal{A})$. A third central result is that functors $\mathcal{F}$ respect reflexivity.

Furthermore, the authors give examples of Δ-equivalent algebras and of Δ-inequivalent algebras. Let a commutative subspace lattice (CSL) be a projection lattice $\mathcal{L}$ of commuting elements. The corresponding algebra $\mathrm{Alg}(\mathcal{L})$ is called a CSL algebra. They are reflexive. If $\mathcal{L}$ is totally ordered then $\mathcal{L}$ is a nest and the algebra $\mathrm{Alg}(\mathcal{L})$ is a nest algebra. The author proves that two CSL algebras are Δ-equivalent if and only if they are TRO-equivalent. Moreover, similar nest algebras have equivalent categories of normal representations.

For related concepts the reader is referred to the publications [D. P. Blecher, P. S. Muhly and V. I. Paulsen, Mem. Amer. Math. Soc. **143** (2000), no. 681, viii+94 pp.; MR1645699 (2000j:46132); G. K. Eleftherakis and V. I. Paulsen, Math. Ann. **341** (2008), no. 1, 99–112; MR2377471 (2009a:46111)].

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.